Playing Games with Quantum Information: Experiments with Photons and Laser-Cooled Atoms

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Outline

The "Superoperator"

- Introduction/Review of quantum information
- Process Tomography with atoms in lattices
- Process Tomography of a quantum logic device with photons
- Two-Photon Switch

Quantum Information

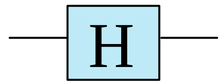
What's so great about it?

- **Information is physical (which is quantum)**
- **Factoring**
- **Searching**
- **Modelling quantum systems**
- **Cryptography**

What is a computer quantum?

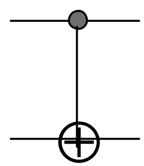
1. Superposition: $|\Psi\rangle = c_0|0\rangle + c_1|1\rangle$ (the qubit)

   Single qubit gates ie. Hadamards, Pi pulses, Waveplates



2. Entanglement: $|\Psi_T\rangle = (c_0|0\rangle|0\rangle + c_1|1\rangle|1\rangle) \neq |\Psi_a\rangle|\Psi_b\rangle$

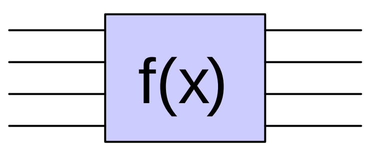
   Two qubit gates ie. Cnot, Kerr Effect, Ion Repulsion



3. Parallelism: $|\Psi\rangle = c_o|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + \ldots$

   $|f(\Psi)\rangle = c_o|f(000)\rangle + c_1|f(001)\rangle + c_2|f(010)\rangle + c_3|f(011)\rangle + \ldots$

   2^n coefficients



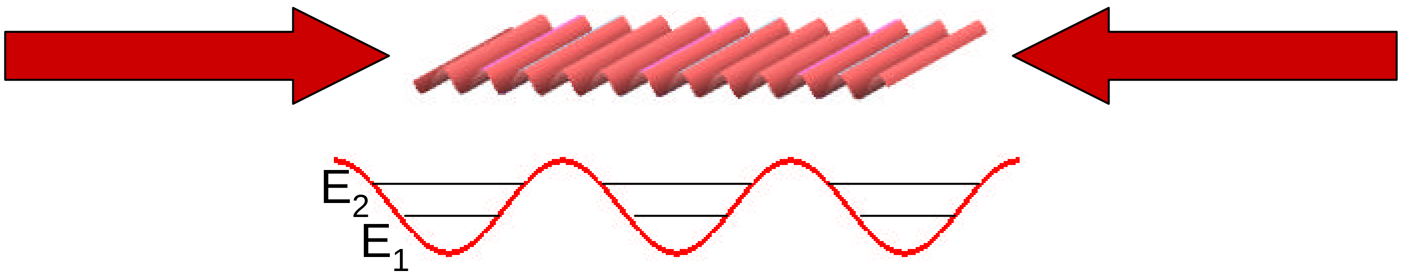
Systems For Quantum Information

Laser-cooled neutral atoms in lattices

$|\Psi\rangle = c_0|E_1\rangle + c_1|E_2\rangle$

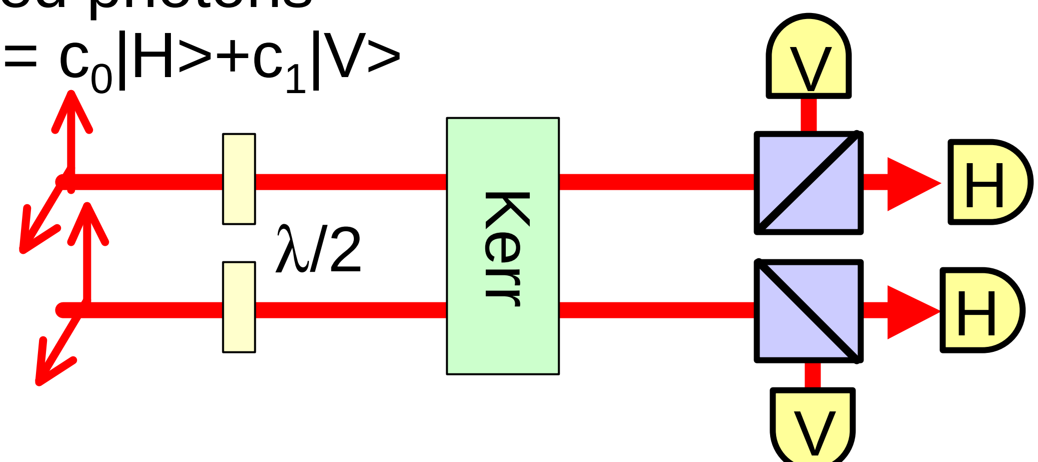
$U \propto p \bullet E \propto$ Intensity

Standing Wave



Polarized photons

$|\Psi\rangle = c_0|H\rangle + c_1|V\rangle$



Problem:
Kerr Effect
is 10^{10}
too small

The Real Problem

- The danger of errors grows exponentially with the size of the quantum system.
- With error-correction there is a threshold for the error-rate above which quantum computation is possible.
- A major goal is to learn to completely characterize the evolution (and decoherence) of physical quantum systems in order to design and adapt error-control systems.
- The tools are "quantum state tomography" and "quantum process tomography": full characterisation of the density matrix or Wigner function, and of the "$uperoperator" which describes its time-evolution.

Density matrices & superoperators

One photon: H or V.
State: two coefficients

$$\Psi = \begin{pmatrix} c_H \\ c_V \end{pmatrix}$$

Density matrix: 2x2=4 coefficients

intensity of horizontal

intensity of 45°
& RH circular.

$$\rho = \begin{pmatrix} c_{HH} & c_{VH} \\ c_{HV} & c_{VV} \end{pmatrix}$$

intensity of vertical

Propagator (superoperator $\mathcal{E}$): 4x4 = 16 coefficients.

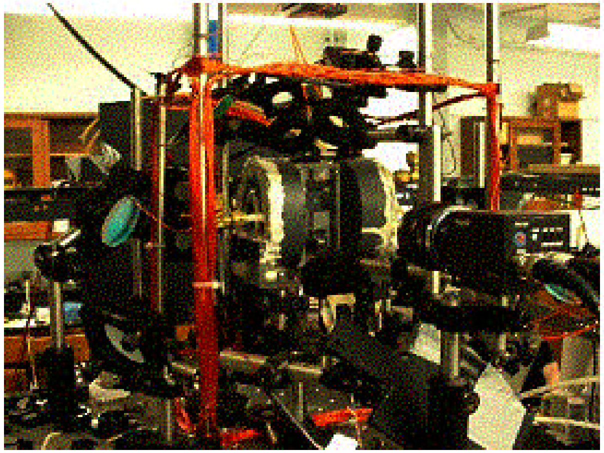
$$\rho' = \varepsilon\{\rho\}$$

Prepare a complete set of input density matrices ρ_i ➡ Make 4 measurements on each to extract ρ_i'

Atoms in Lattices

Quantum Process Tomography: determines the "real-world" operation
– including decoherence and losses

ρ_{in} $\longrightarrow$



$\longrightarrow$ ρ_{out}

"Black Box"

$|0\rangle\langle 0|$

$P_0|0\rangle\langle 0| + P_1|1\rangle\langle 1|$

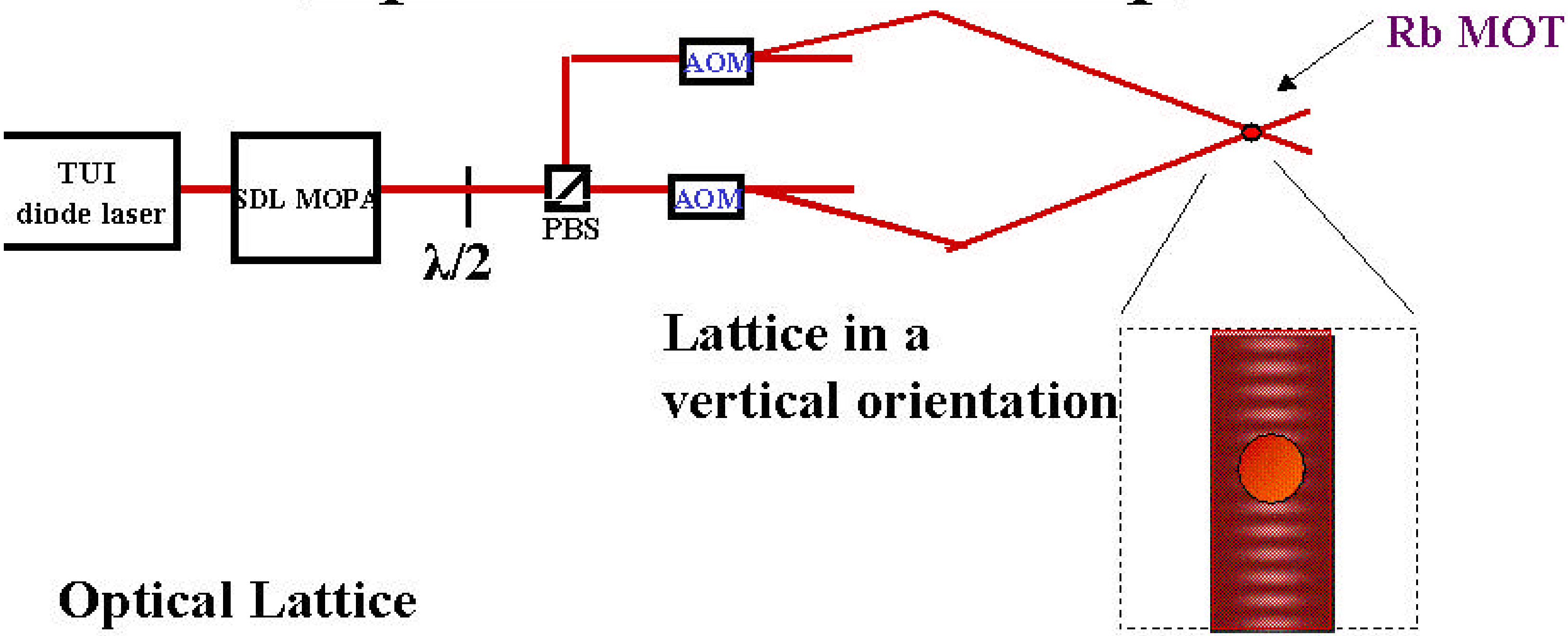
$(a|0\rangle + b|1\rangle)(\ldots)$

$(a|0\rangle + ib|1\rangle)(\ldots)$

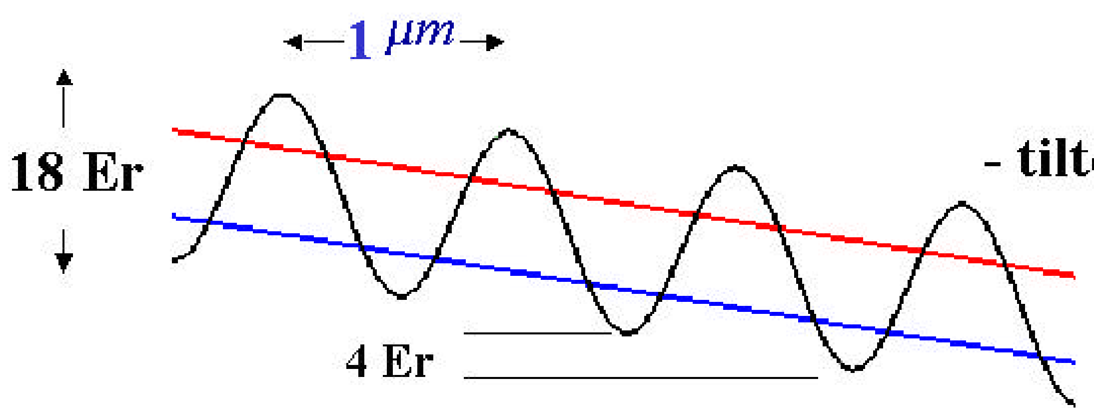
Process $\mathcal{E}$
desired +decoherence+loss
errors

Measure Outputs
(State tomography)

Optical Lattice Setup



Optical Lattice



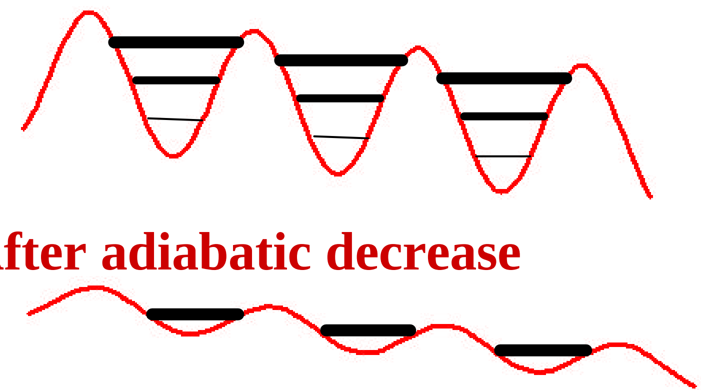
- tilted by gravity

State Tomography in an Optical Lattice

Measuring State Populations

Thermal

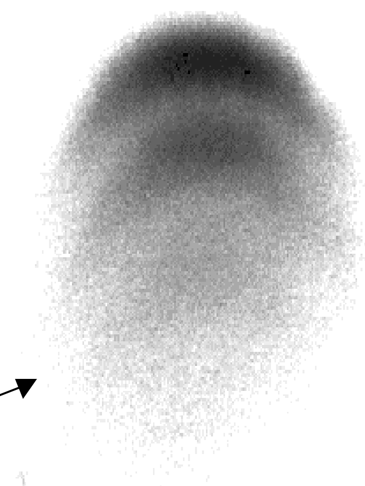
Initial Lattice

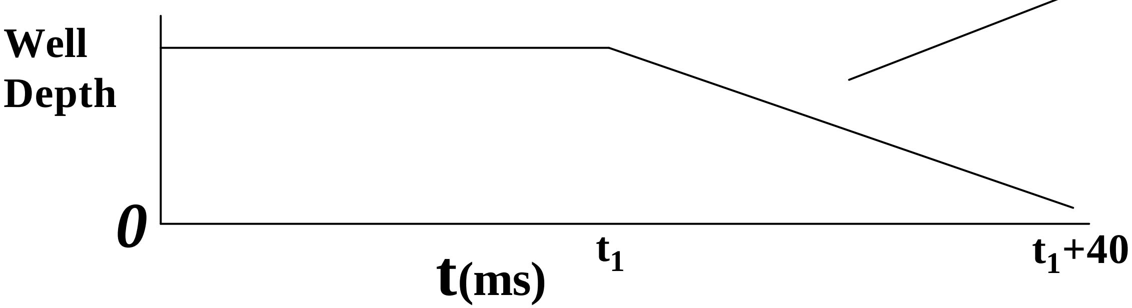


After adiabatic decrease

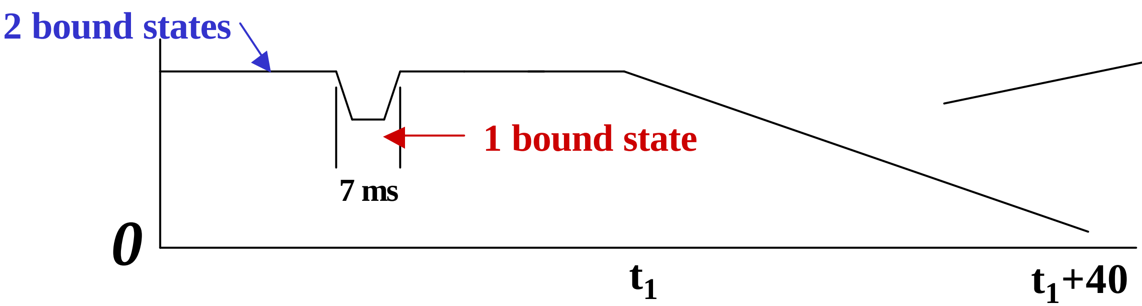
Ground State

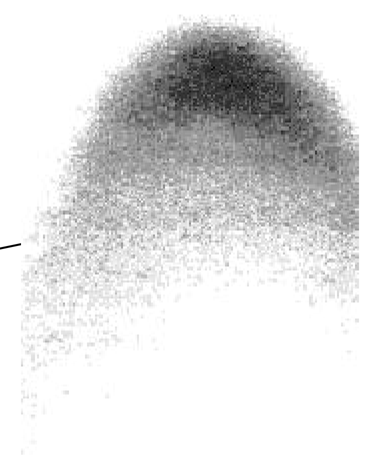
Excited State





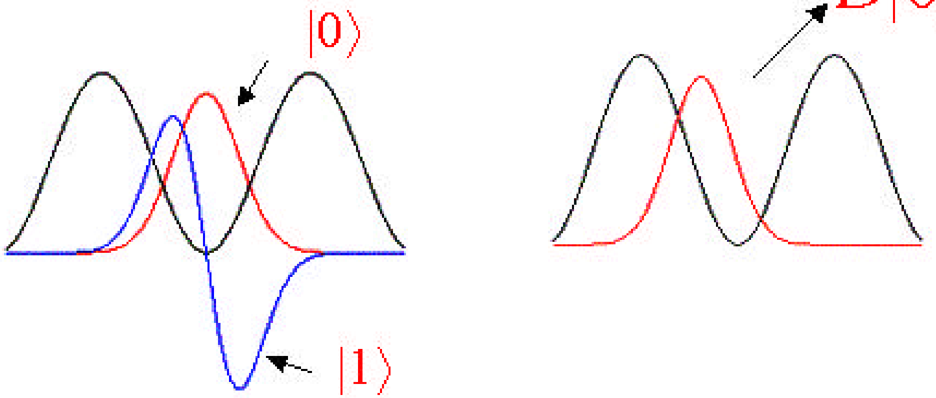
Preparing a ground state





State tomography continued

Displacement



$D|0\rangle \approx a|0\rangle + b|1\rangle +$ **unbound atoms**

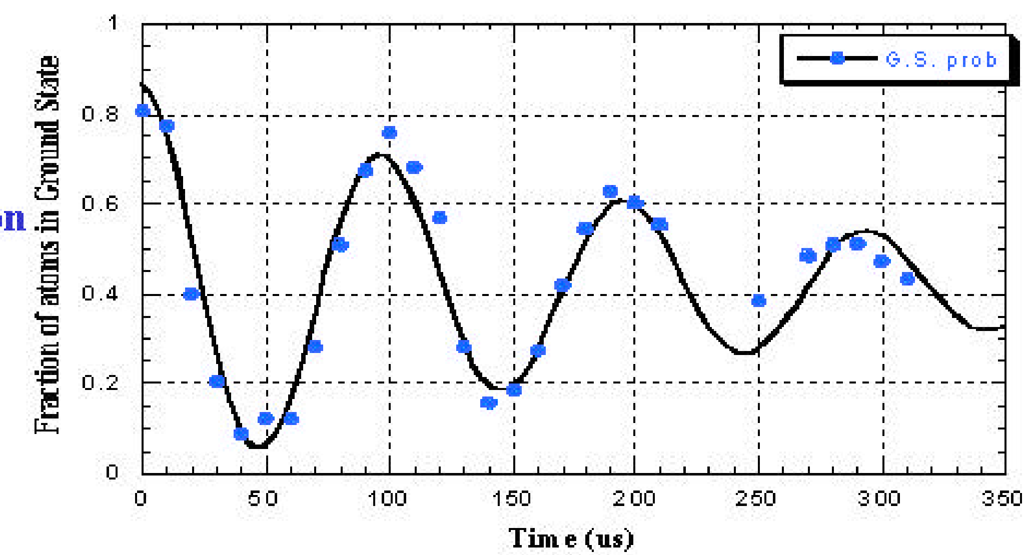
$$\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} \rightarrow \begin{bmatrix} a & b \\ -b^* & a \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

measured

Oscillation period

Displace ground state
Time delay (phase evolution)
Undo displacement
Measure ground state population

Ground state population vs time delay



Performing State Tomography

Project the unknown state onto a set of known states.

$$\langle \Phi_i | \rho | \Phi_i \rangle = \langle 0 | \hat{U}^{\dagger} \rho \hat{U} | 0 \rangle \qquad |\Phi\rangle = \hat{R}(\omega t)\hat{D}(d)|0\rangle$$

E.g., state tomography on $0.8|0\rangle + 0.6i|1\rangle$

Measurement Basis

$\lvert 0\rangle$	$0.9\lvert 0\rangle + 0.4\lvert 1\rangle$	$0.9\lvert 0\rangle + i0.4\lvert 1\rangle$
vs $\lvert 1\rangle$	**vs** $0.4\lvert 0\rangle - 0.9\lvert 1\rangle$	**vs** $0.4\lvert 0\rangle - 0.9i\lvert 1\rangle$

ρ_{11}

ρ_{22}

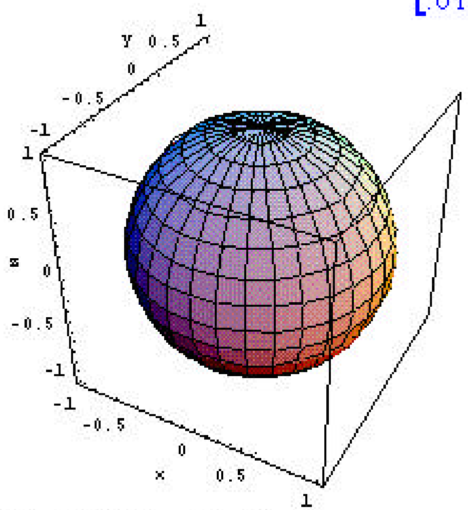
$\rightarrow \rho_{12} + \rho_{12}^*$

$\rightarrow \rho_{12} - \rho_{12}^*$

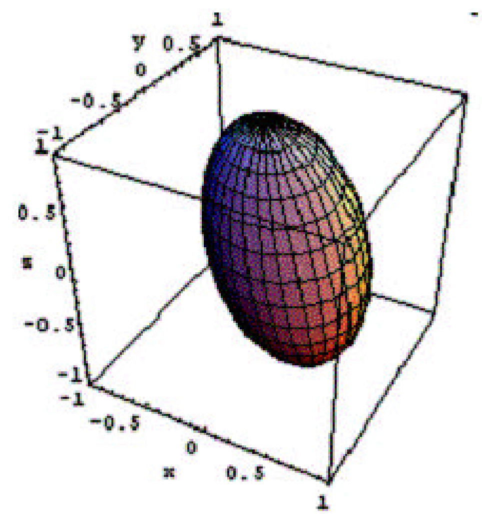
QPT of Decoherence

Operation:
Sitting in the lattice
for 1 period.

ρ_{in}		ρ_{out}
$\begin{bmatrix} .9 & -.01+.02\,I \\ -.01-.02\,I & .1 \end{bmatrix}$		$\begin{bmatrix} .9 & -.01 \\ -.01 & .1 \end{bmatrix}$
$\begin{bmatrix} .60 & .01+.02\,I \\ .01-.02\,I & .40 \end{bmatrix}$		$\begin{bmatrix} .60 & .05-.02I \\ .05+.02I & .40 \end{bmatrix}$
$\begin{bmatrix} .69 & .41+.010\,I \\ .41-.010\,I & .31 \end{bmatrix}$	$\rightarrow$	$\begin{bmatrix} .69 & .28-.010I \\ .28+.010I & .31 \end{bmatrix}$
$\begin{bmatrix} .69 & .010-.35\,I \\ .010+.35\,I & .31 \end{bmatrix}$		$\begin{bmatrix} .69 & 0.-.26\,I \\ 0.+.26\,I & .31 \end{bmatrix}$

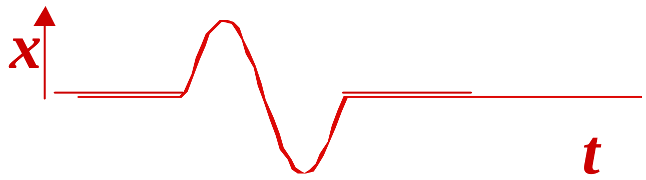


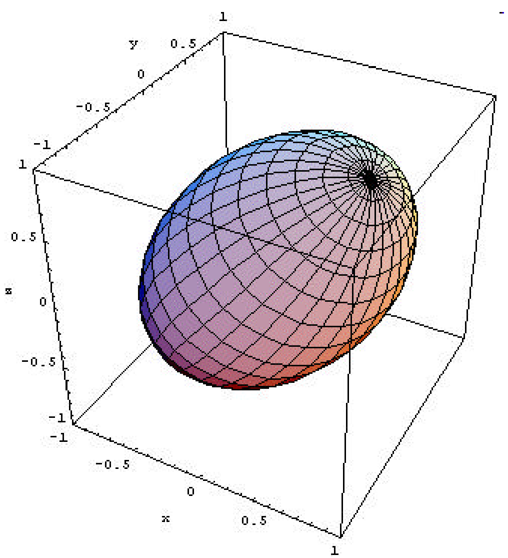
Initial Bloch Sphere



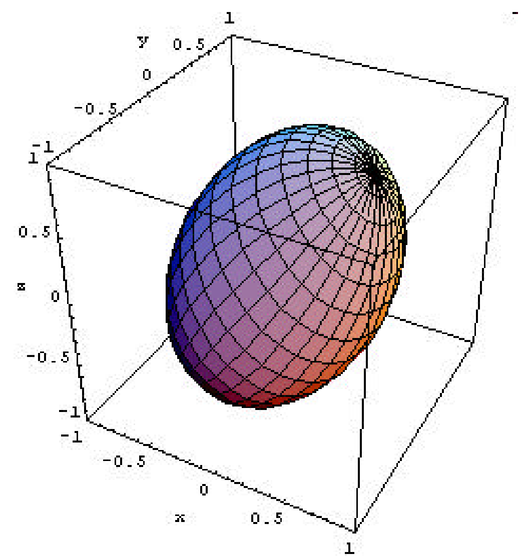
QPT of Driving Oscillations

Operation: Resonantly shake the lattice.





Observed Bloch Sphere



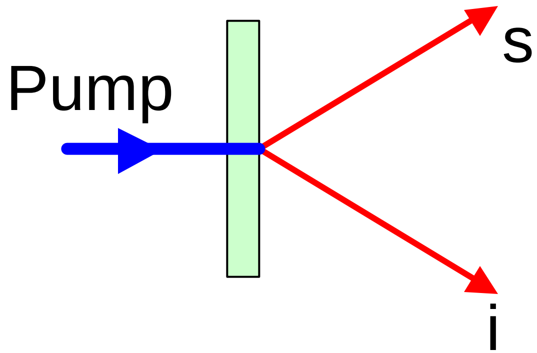
Modelled Bloch Sphere from theory
(Harmonic oscillator plus decoherence
from previous measurement)

What about two-qubit processes?

- **The lattice experiment can do process tomography on any single-qubit process.**
- **Two-particle logic gates and processes:**
  Polarized photon pairs from spontaneous parametric downconversion.
- **Now each density operator has 4x4=16 elements and the superoperator has 16x16=256 elements.**
- **Measurement Procedure:**
  Prepare a complete set of 16 r
  For each r **measure: H-H, H-V, H-RHC, V-45°, etc.**

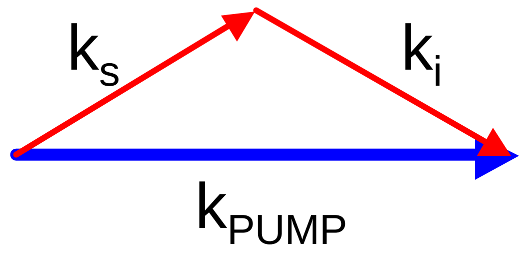
Spontaneous Parametric Downconversion

Downconversion

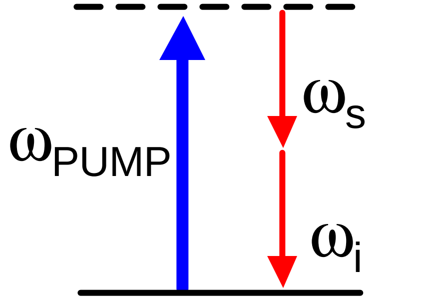


- A pump photon is spontaneously converted into two lower frequency photons in a material with a nonzero $\chi^{(2)}$

Momentum is conserved..



..as well as energy

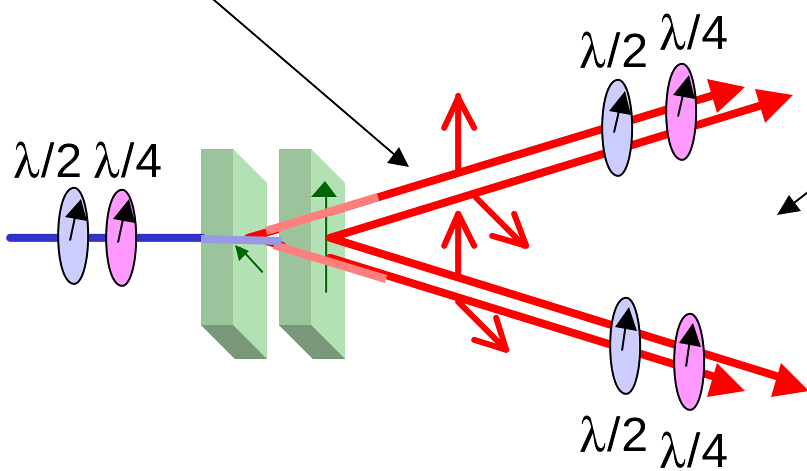


$$\varphi_{PUMP} = \varphi_s + \varphi_i$$

General Two-Photon State Production

$|\Psi\rangle = c_1|H\rangle|H\rangle + c_2 e^{i\phi}|V\rangle|V\rangle$

$|\Psi\rangle = c_1|H\rangle|H\rangle + c_2 e^{i\phi 2}|H\rangle|V\rangle + c_3 e^{i\phi 3}|V\rangle|H\rangle + c_4 e^{i\phi 4}|V\rangle|V\rangle$

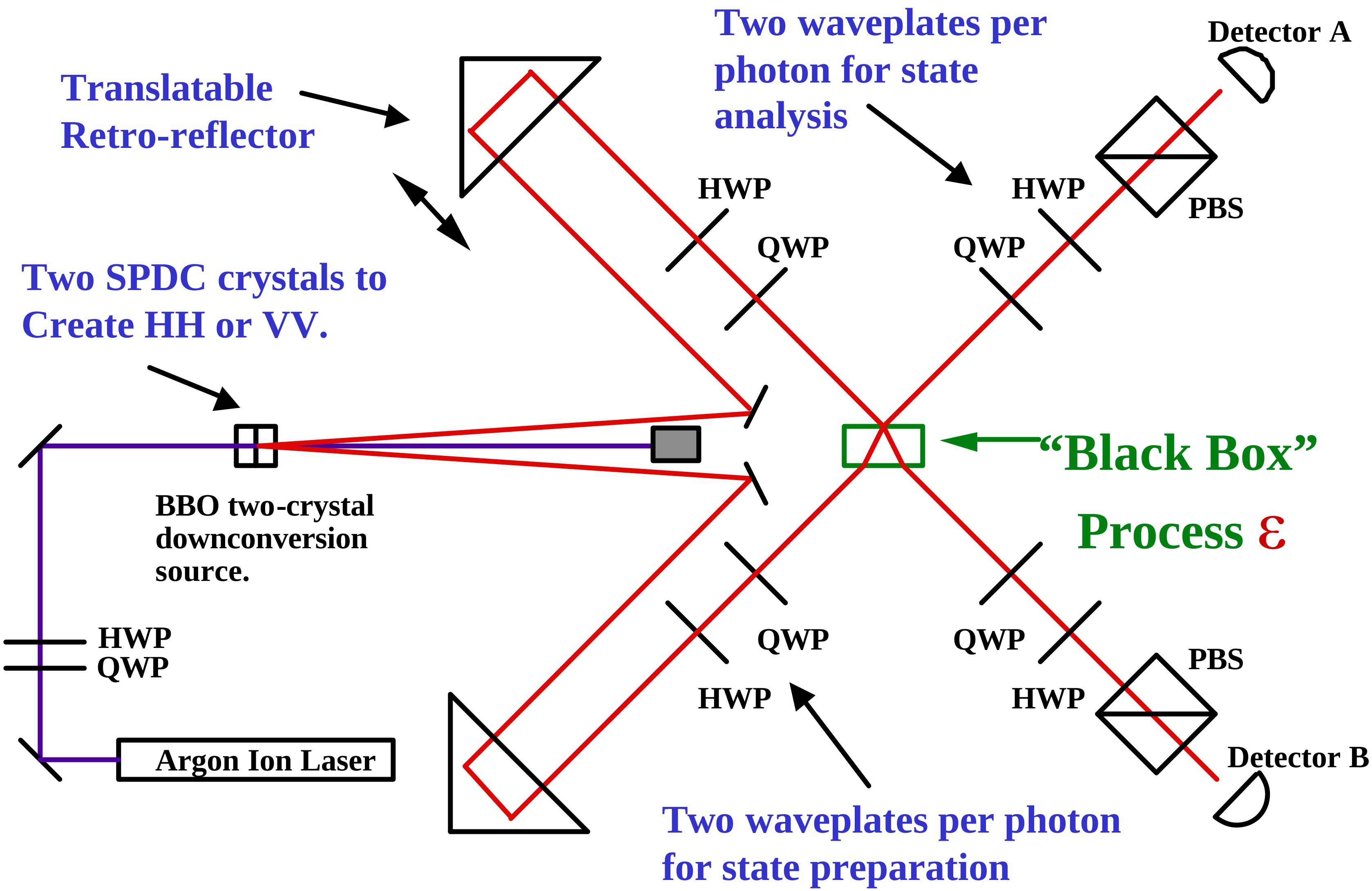


ie. With just one λ/2 we can create the singlet state

$|\Psi\rangle = |H\rangle|V\rangle - |V\rangle|H\rangle$

- By adjusting 6 waveplates we can produce a complete set of input states to measure the superoperator

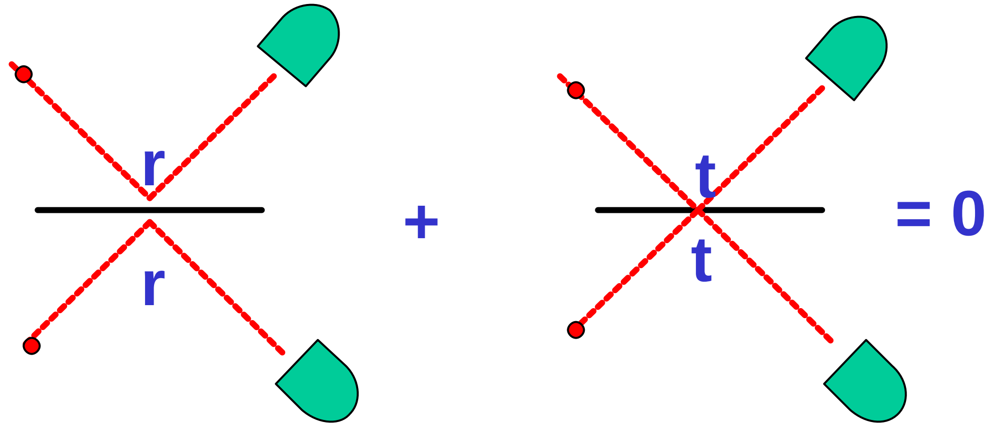
Two-photon Process Tomography



Our Black Box

The (not-so) simple
50/50 beamsplitter

Codename:
Bell-state Filter

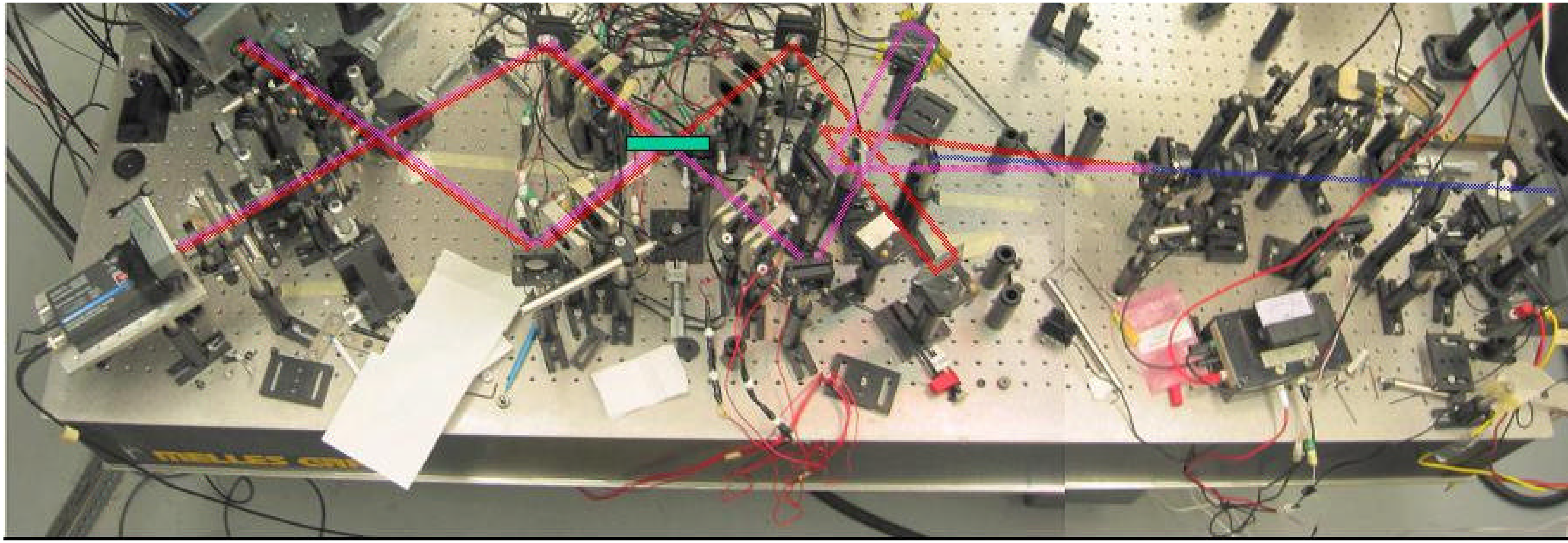


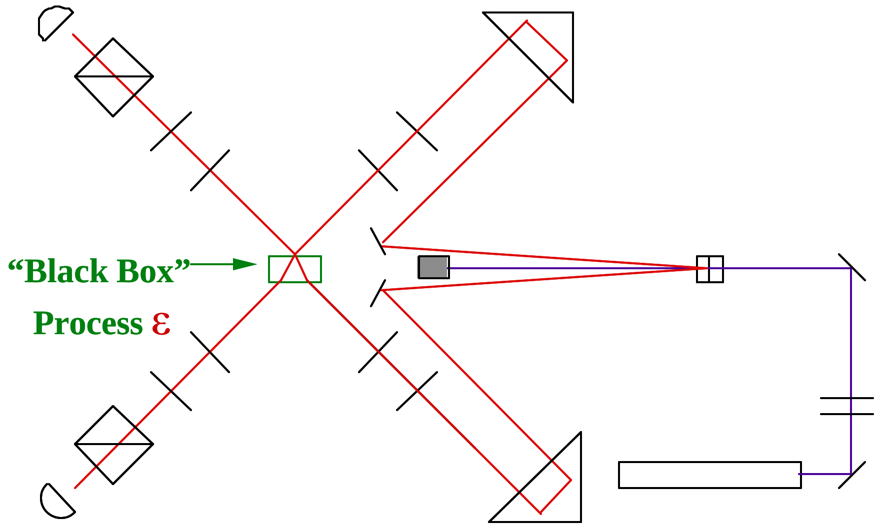
Bell - State	Coincidence Counts
$\|\Phi^{+}\rangle = \|HH\rangle + \|VV\rangle$	**No (symmetric)**
$\|\Phi^{-}\rangle = \|HH\rangle - \|VV\rangle$	**No (symmetric)**
$\|\Psi^{+}\rangle = \|HV\rangle + \|VH\rangle$	**No (symmetric)**
$\|\Psi^{-}\rangle = \|HV\rangle - \|VH\rangle$	**Yes! (anti-symmetric)**

Uses: Quantum Teleportation, Quantum Repeaters, CNOT

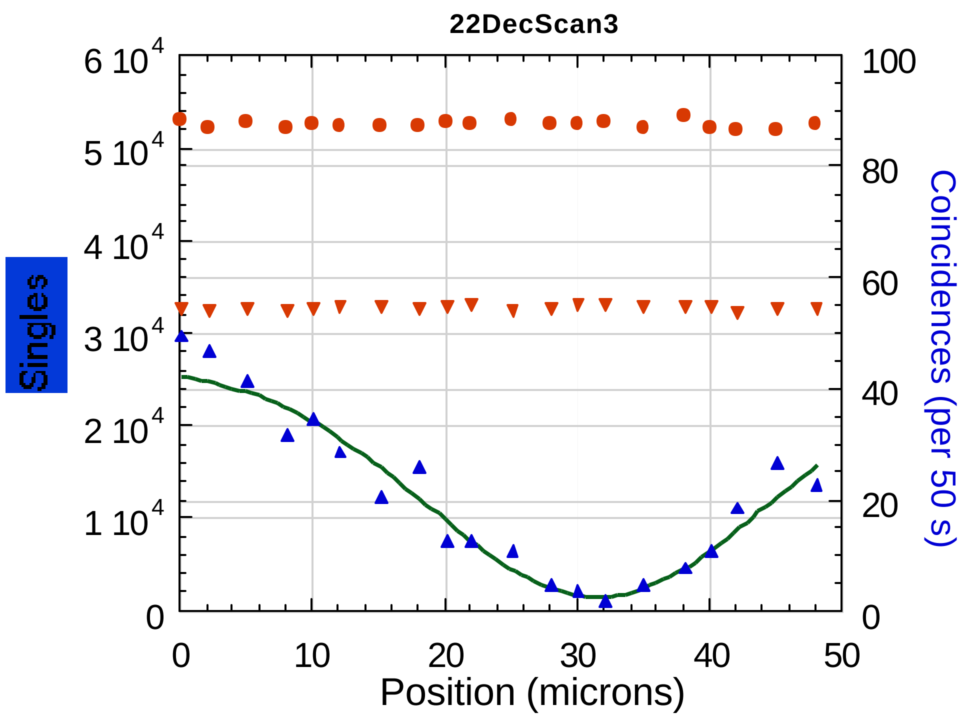
Our Goal: use process tomography to test this filter.

Process Tomography Apparatus





Hong-Ou-Mandel Interference



> 85% visibility for HH and VV polarizations

HOM acts as a filter for the Bell state:

$\Psi^{-} = (HV\text{-}VH)/\sqrt{2}$

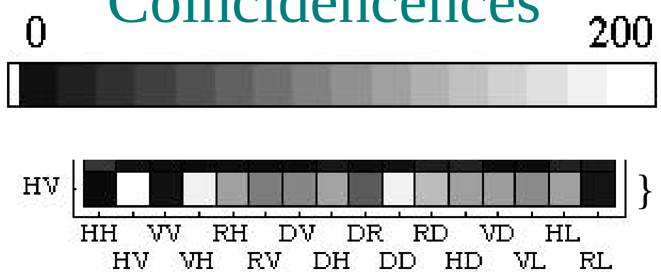
Goal: Use Quantum Process Tomography to find the *superoperator* which takes $\rho_{in} \rightarrow \rho_{out}$

Measuring the superoperator

1. Input: $|\Psi\rangle = |H\rangle\,|V\rangle$

2. Measurement

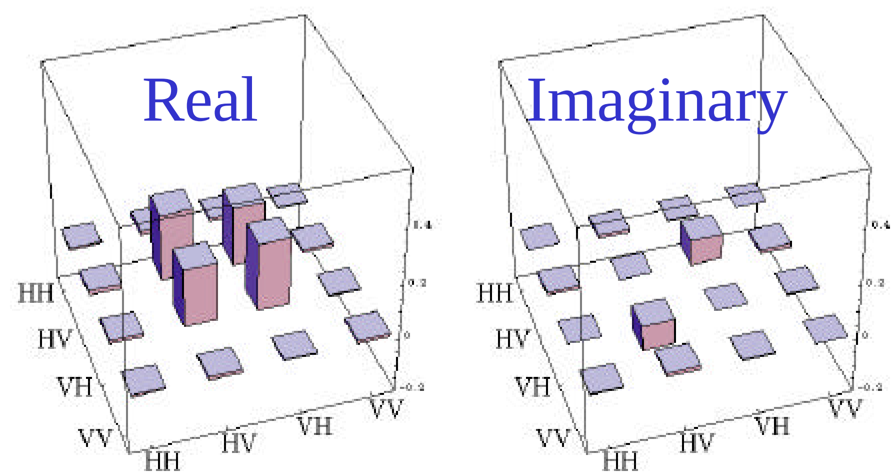
Coincidencences



16 analyzer settings

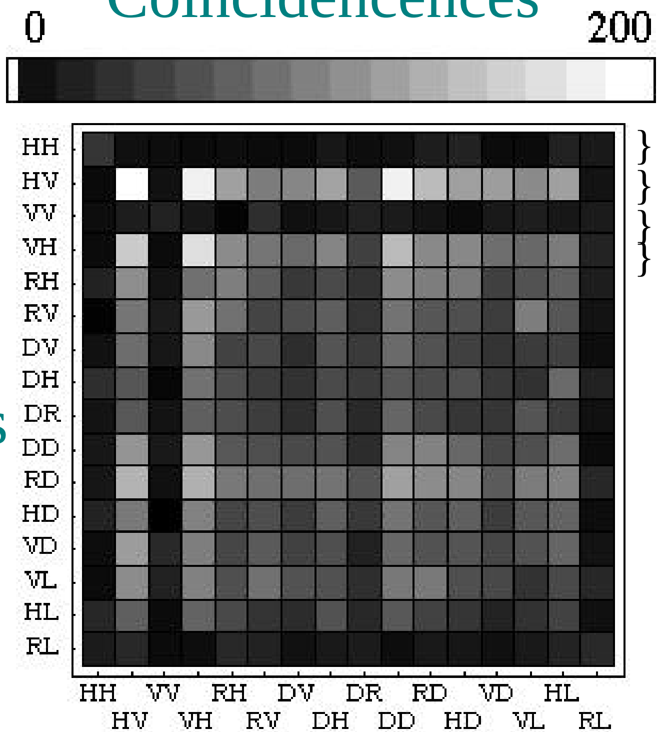
3. Output:

$\rho_{out} =$



Repeat for 16 Input States

Coincidences



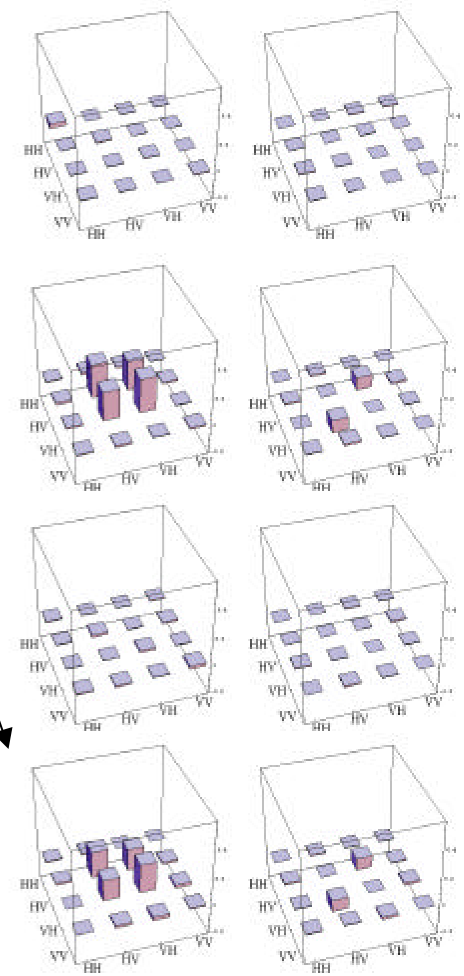
16 input states

16 analyzer settings

etc.

Output ρ

Input



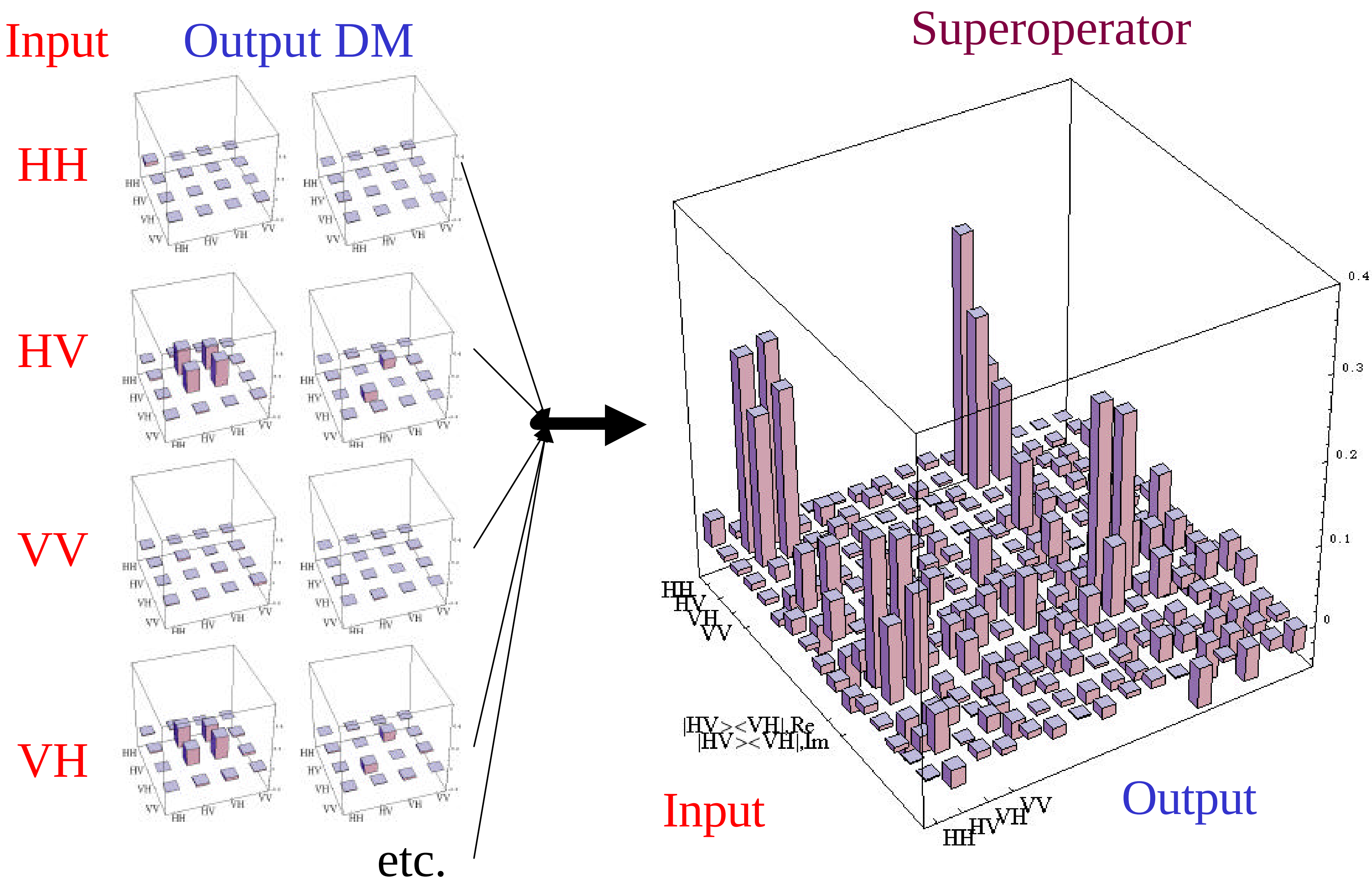
HH

HV

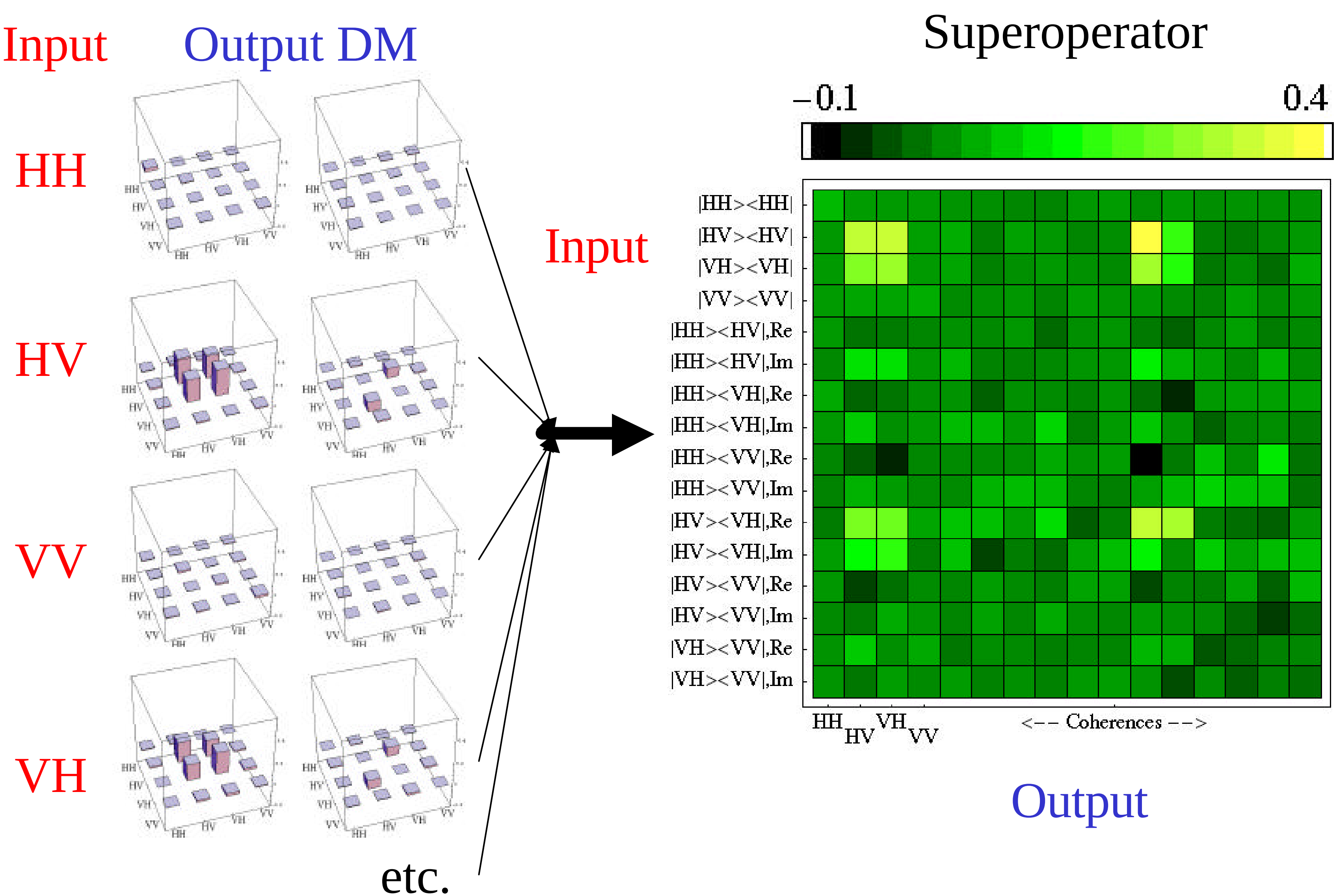
VV

VH

The Urban Representation

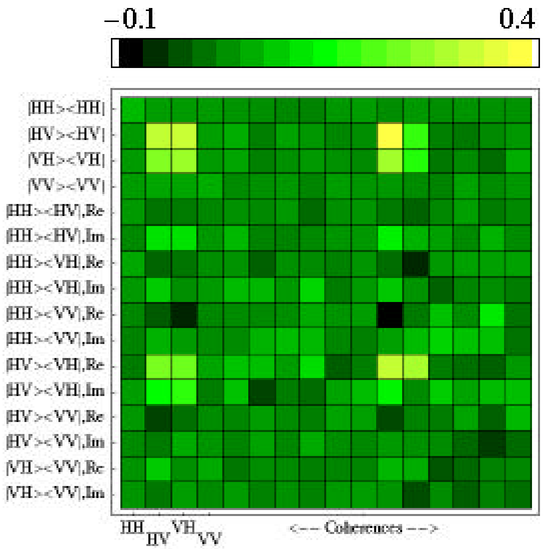


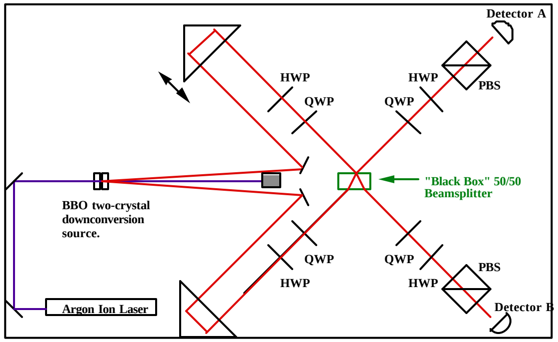
The PEI Representation

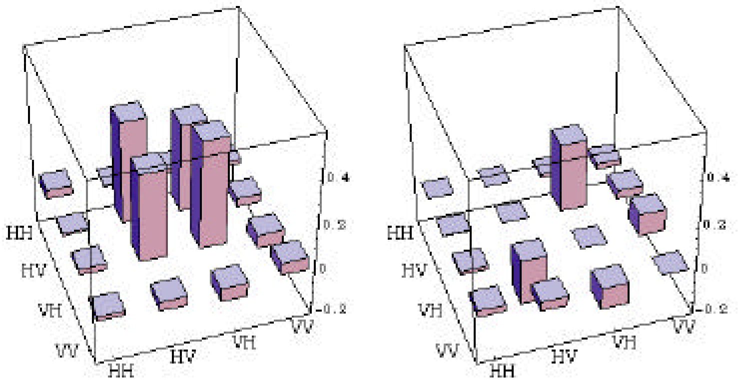


Testing the superoperator

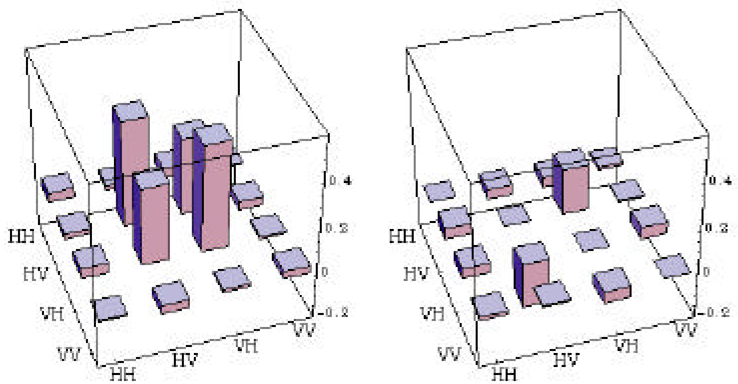
LL = input state







Predicted $N_{photons} = 297 \pm 14$

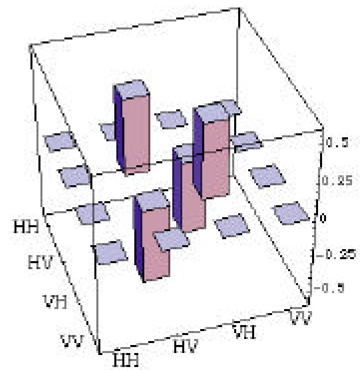


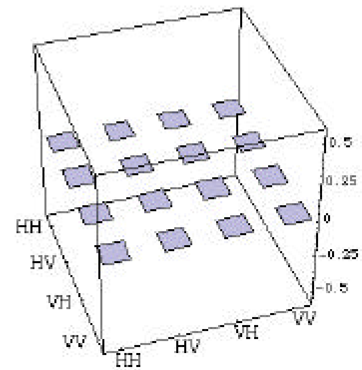
Observed $N_{photons} = 314$

So, How's Our Bell-State Filter?

In: Bell singlet state: Ψ^{-} = (HV-VH)/v2

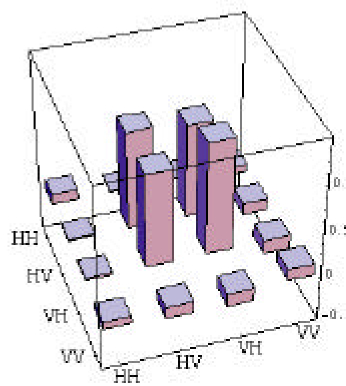
$$\Psi^{-} = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix} =$$

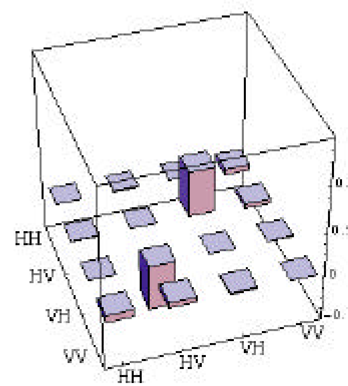




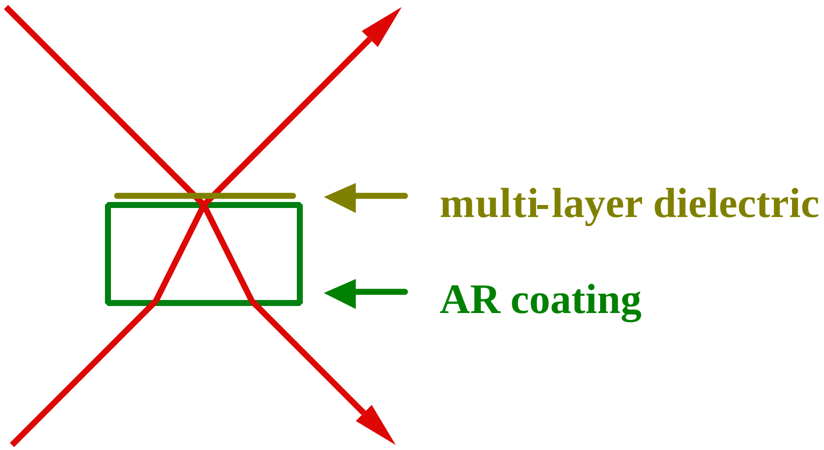
Out: $\neq \Psi^{-}$, but is a different maximally entangled state:

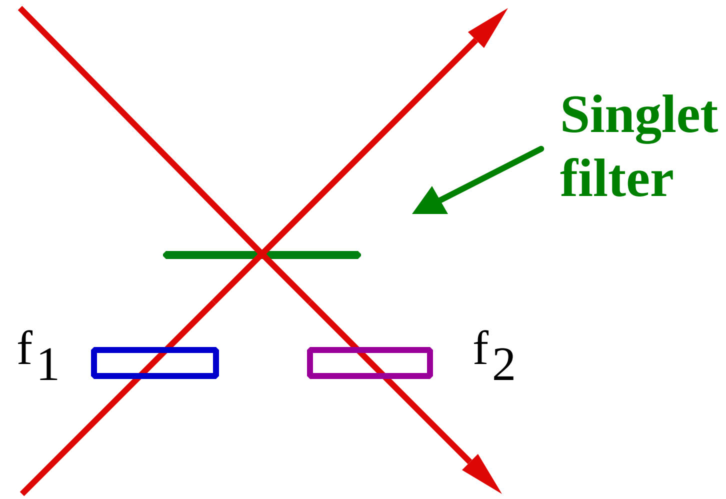
Ψ^{-} 1





Model of real-world beamsplitter





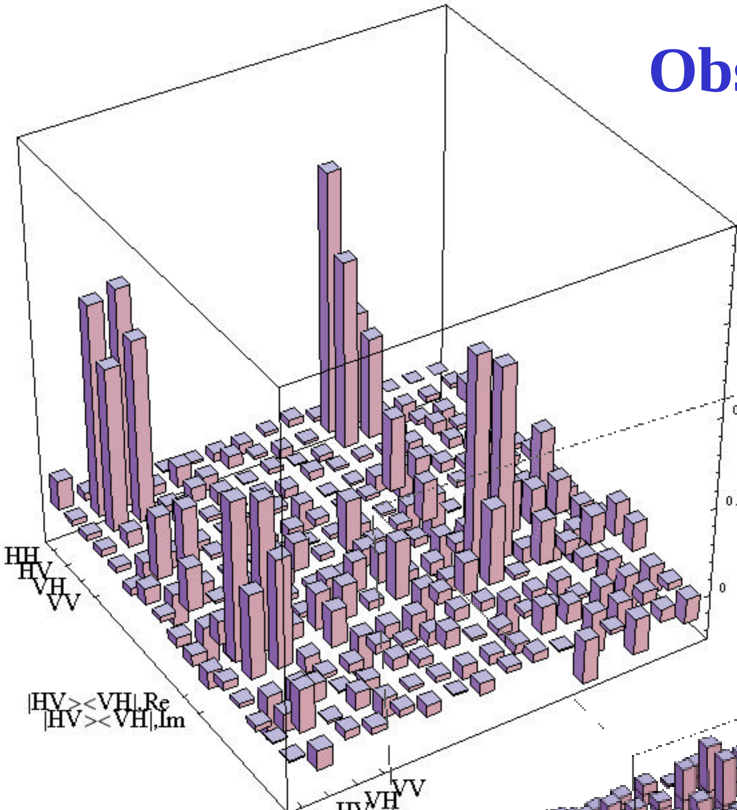
45° "unpolarized" 50/50 dielectric beamsplitter at 702 nm (CVI Laser)

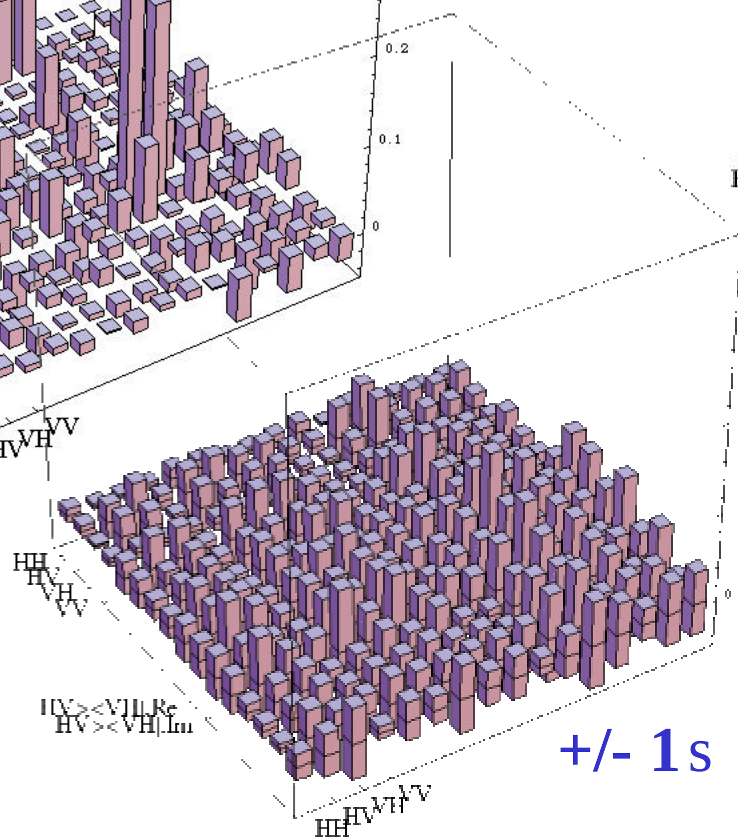
birefringent element
+
singlet-state filter
+
birefringent element

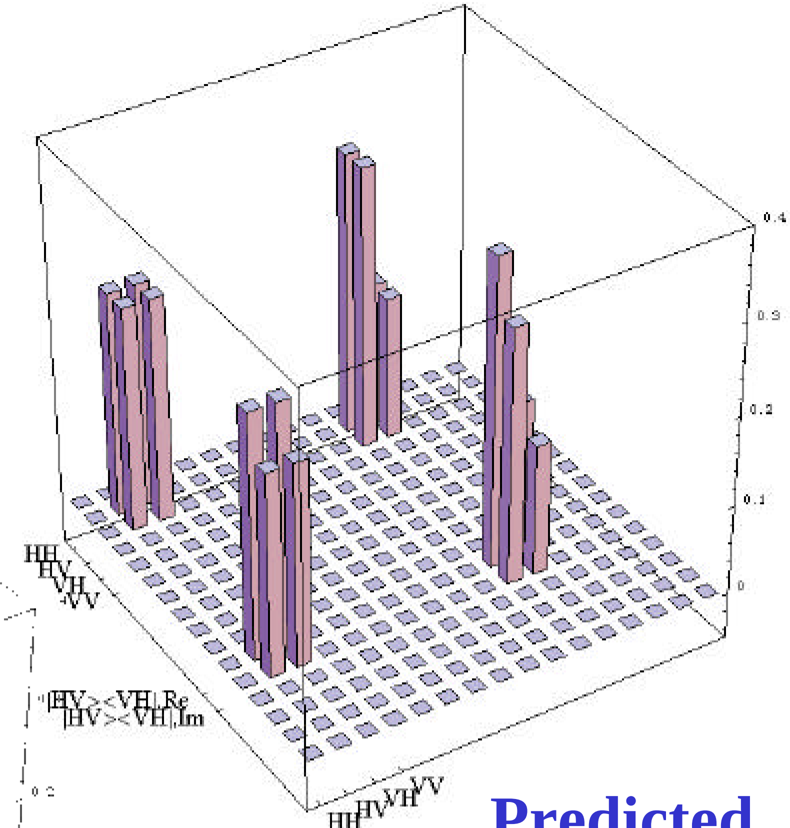
Best Fit: $\phi_1 = 0.76$ p
$\phi_2 = 0.80$ p

Comparison to measured Superop

Observed







Predicted

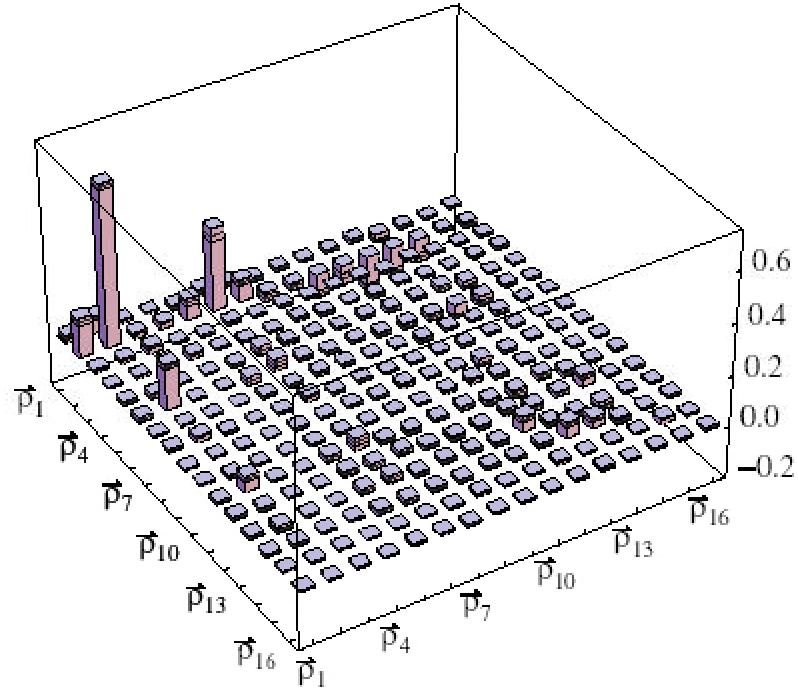
$\chi^2 = 477$

N-d.f. = 253

+/- 1 s (statistical)

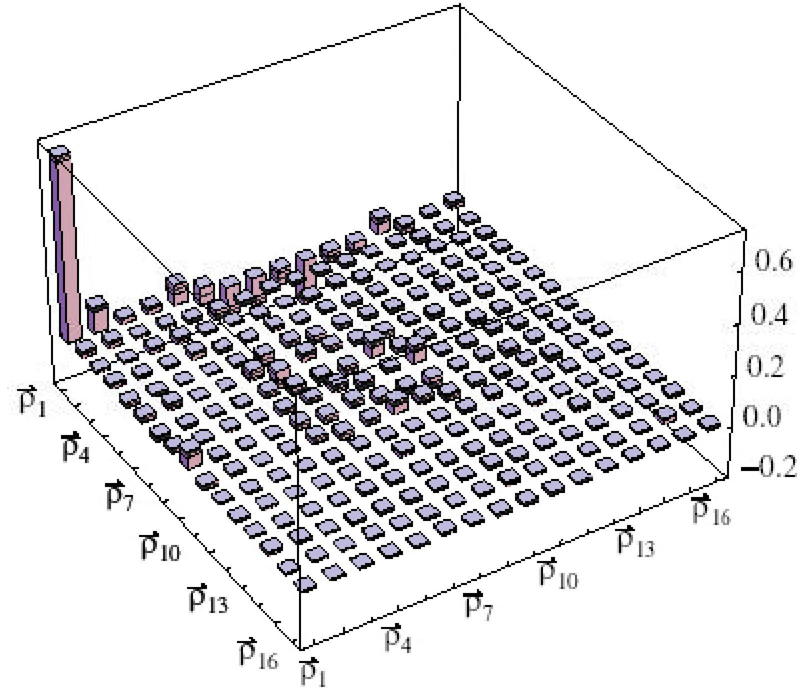
Comparison to ideal filter

Measured superoperator, in Bell-state basis:



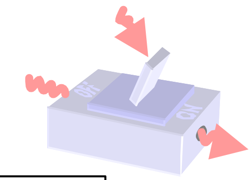
A singlet-state filter would have a single peak, indicating the one transmitted state.

Superoperator after transformation to correct polarisation rotations:

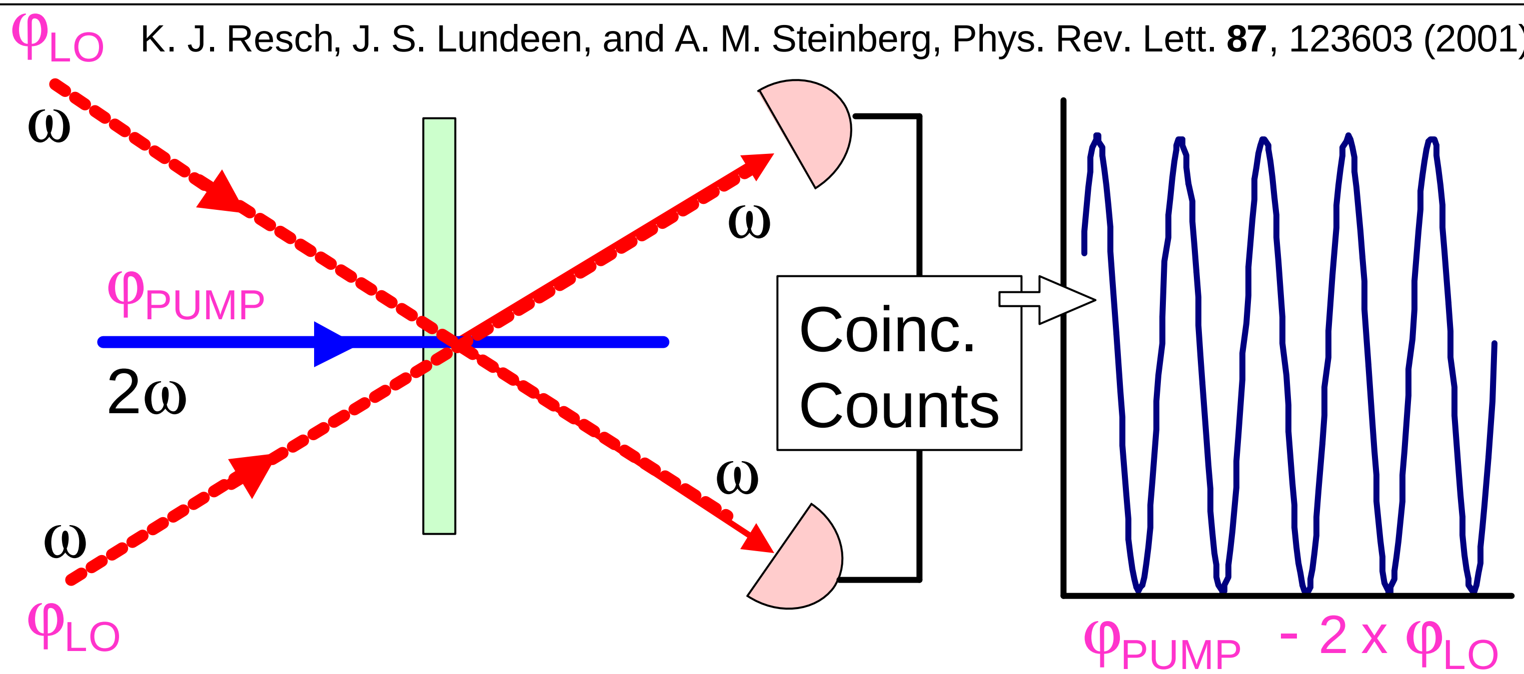


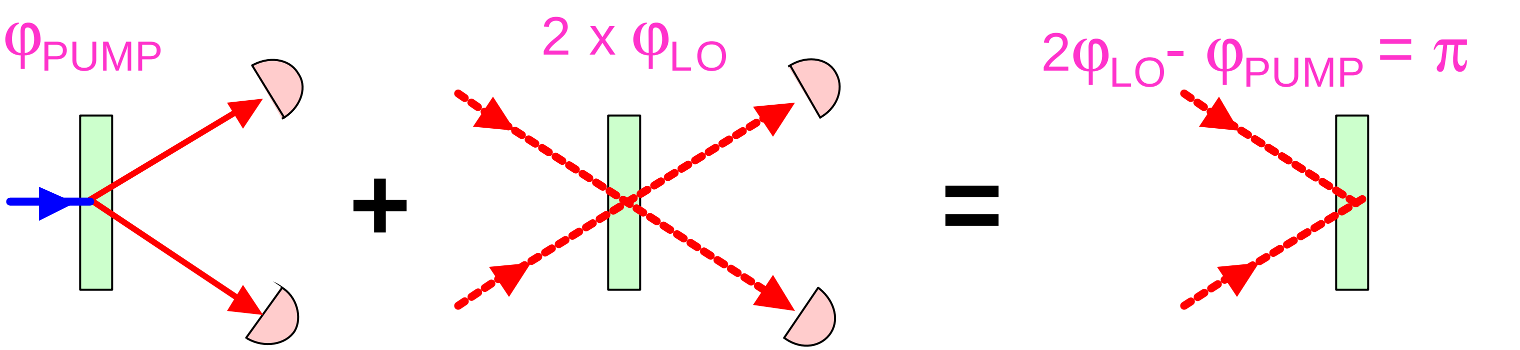
Dominated by a single peak; residuals allow us to estimate degree of decoherence and other errors.

The Switch



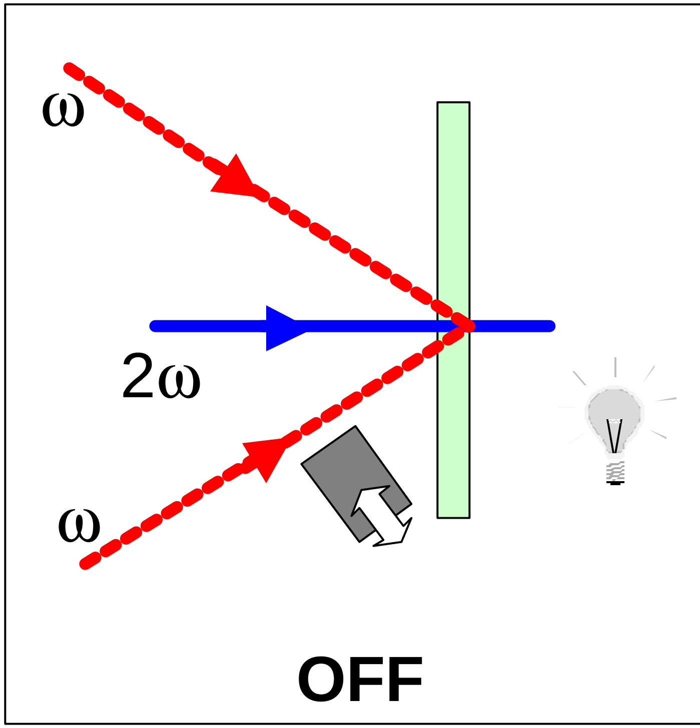
K. J. Resch, J. S. Lundeen, and A. M. Steinberg, Phys. Rev. Lett. **87**, 123603 (2001).





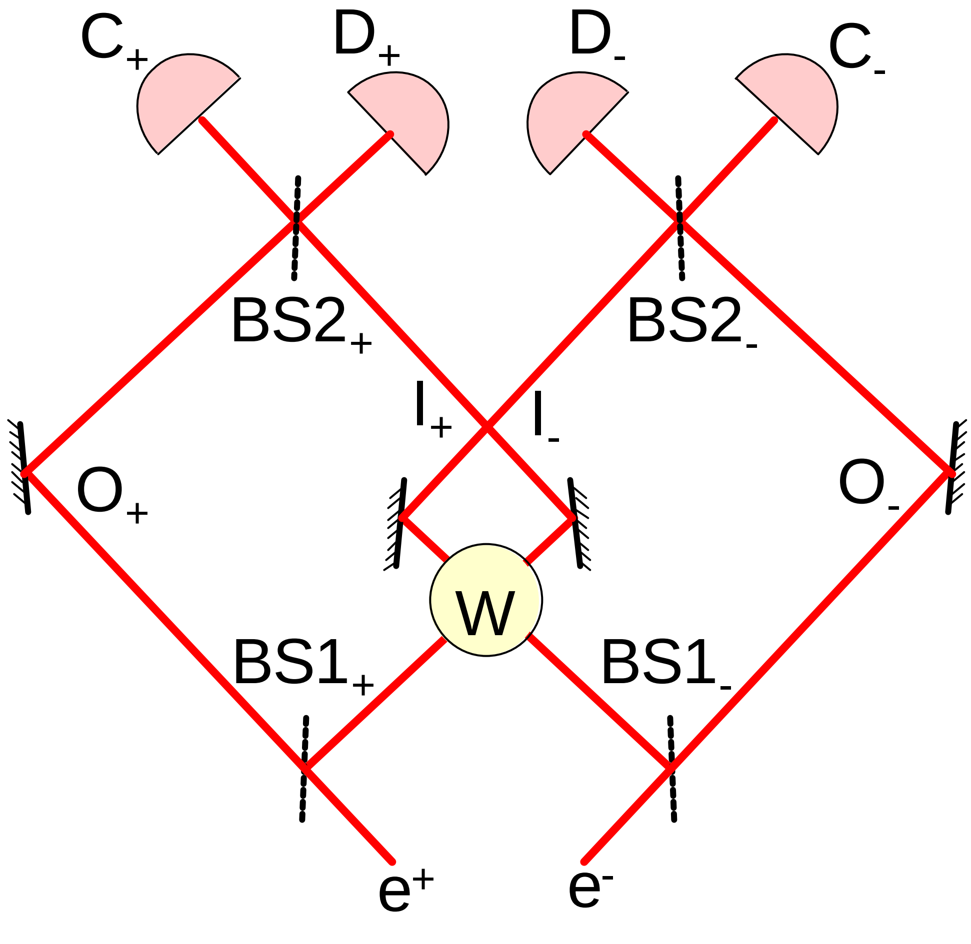
The Switch

•Phase chosen so that coincidences are eliminated



Hardy's Paradox

L. Hardy, Phys. Rev. Lett. **68**, 2981 (1992)



Outcome	Prob
D_+ and C_-	1/16
D_- and C_+	1/16
C_+ and C_-	9/16
D_+ and D_-	1/16
Explosion	4/16

SUMMARY

ATOMS

- State Tomography in a 2 bound-state lattice
- Quantum process tomography: Superoperator for "natural" decoherence and single qubit rotations

PHOTONS

- Quantum process tomography for two polarized photons
- Superoperator for a not so perfect Bell-state filter

THE SWITCI

- Quantum interference allows huge enhancements of optical nonlinearities. Useful for quantum computation?
- Two-photon switch useful for studies of quantum weirdness (Hardy's paradox, weak measurement,…)