# Tailored Quantum Error Correction

## Experimental Effort

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PHOTONICS RESEARCH ONTARIO

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# The Goal



- A major goal is to experimentally completely characterize the evolution (and decoherence) in a quantum system in order to tailor error-control to that particular physical system.
- The tools are "quantum state tomography" and "quantum process tomography": full characterisation of the density matrix or Wigner function, and of the "$uperoperator" which describes its time-evolution.
- Feedback – for adaptive identification of optimal error control strategies.

## Our physical systems:

Polarized **PHOTONS** and **ATOMS** in a lattice

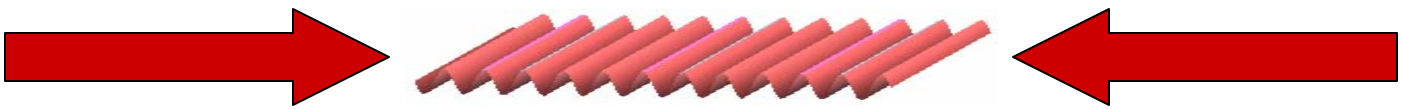
# Systems For Quantum Information

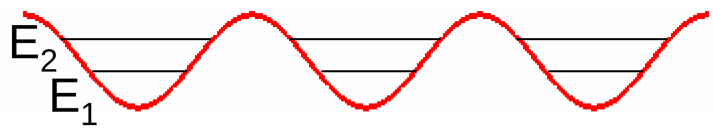
## Laser-cooled neutral atoms in lattices

$|\Psi\rangle = c_0|E_1\rangle + c_1|E_2\rangle$

**$U \propto p \bullet E \propto$ Intensity**

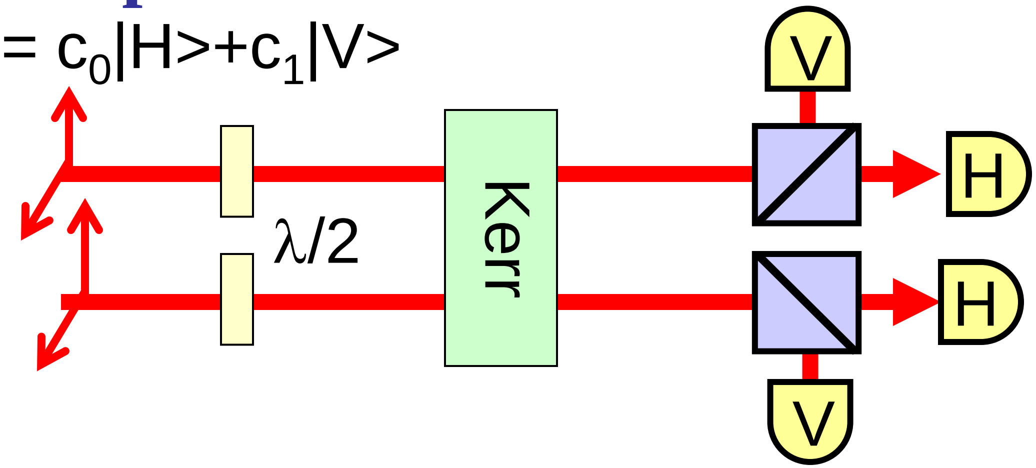
**Standing Wave**






## Polarized photons

$|\Psi\rangle = c_0|H\rangle + c_1|V\rangle$




**Problem:**
**Kerr Effect**
**is $10^{10}$**
**too small**

# Density Matrices & Superoperators



**One photon: H or V.**
**State: two coefficients**

$$\Psi = \begin{pmatrix} c_H \\ c_V \end{pmatrix}$$

**Density matrix: 2x2=4 coefficients**

$$\rho = \begin{pmatrix} c_{HH} & c_{VH} \\ c_{HV} & c_{VV} \end{pmatrix}$$

- intensity of horizontal → $c_{HH}$
- intensity of 45° & RH circular. → $c_{VH}$
- intensity of vertical → $c_{VV}$

**Propagator (superoperator $\varepsilon$): 4x4 = 16 coefficients.**

$$\rho' = \varepsilon\{\rho\}$$

Prepare a complete set of input density matrices $\rho_i$ ➡ Make 4 measurements on each to extract $\rho_i'$

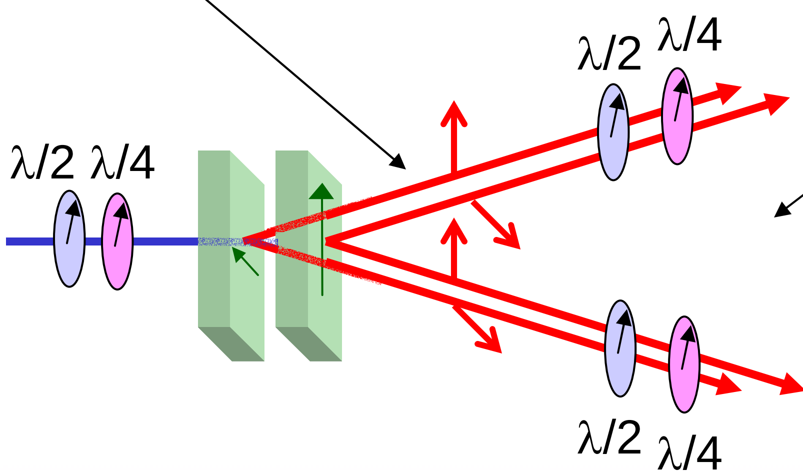
# Process Tomography: Two Photons



- A polarized two-photon state has a 16 element $\rho$
  → We need to make 16 input states and make 16 measurements for each to measure the superoperator $\varepsilon$
- By adjusting the 6 waveplates in the setup below we can produce a complete set of input states

$|\Psi\rangle = c_1|H\rangle|H\rangle + c_2 e^{i\phi}|V\rangle|V\rangle$

$|\Psi\rangle = c_1|H\rangle|H\rangle + c_2 e^{i\phi 2}|H\rangle|V\rangle + c_3 e^{i\phi 3}|V\rangle|H\rangle + c_4 e^{i\phi 4}|V\rangle|V\rangle$

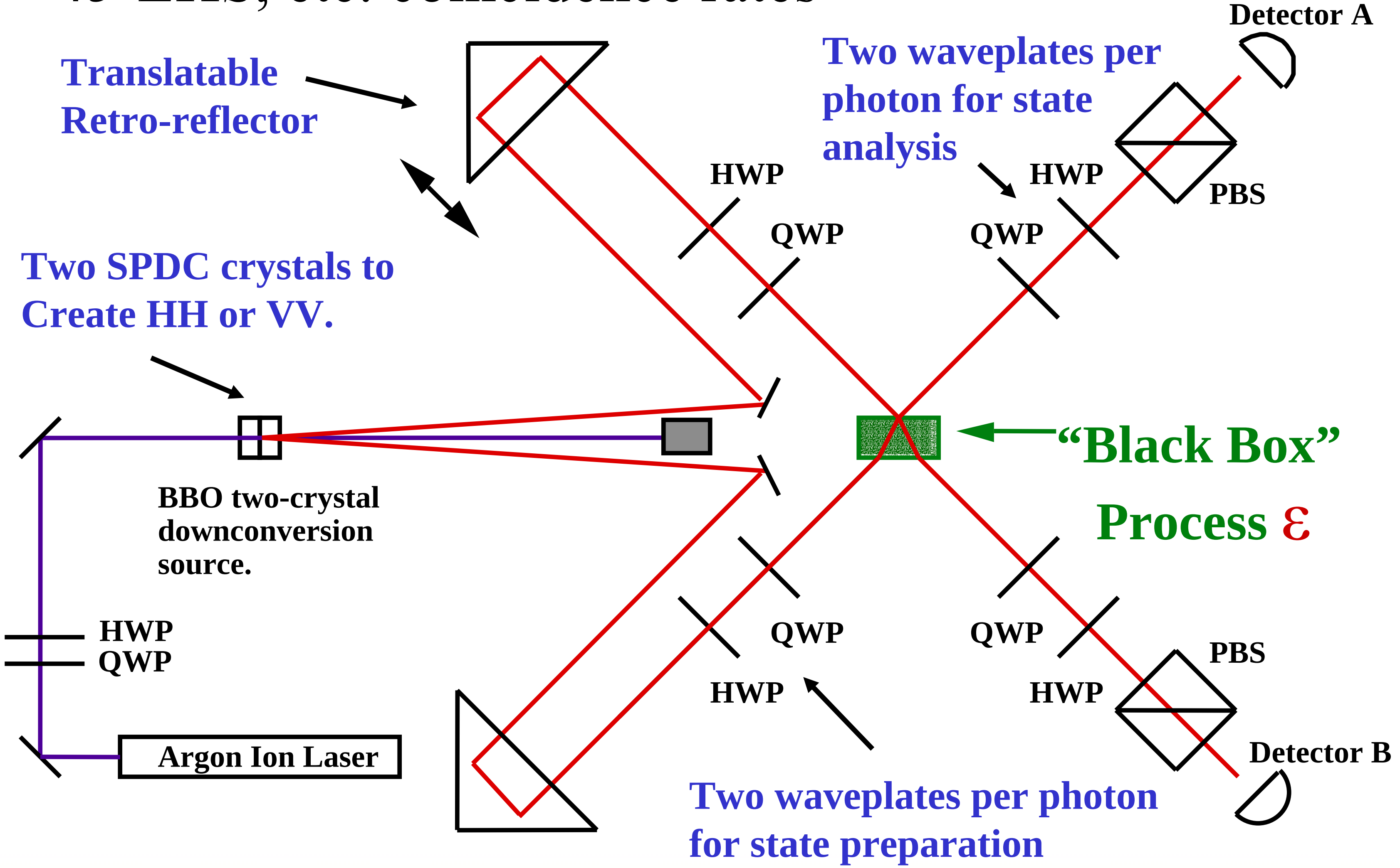



White, James, Eberhard and Kwiat, Phys. Rev. Lett. **83**, 3103 (1999).

# Two-photon Process Tomography



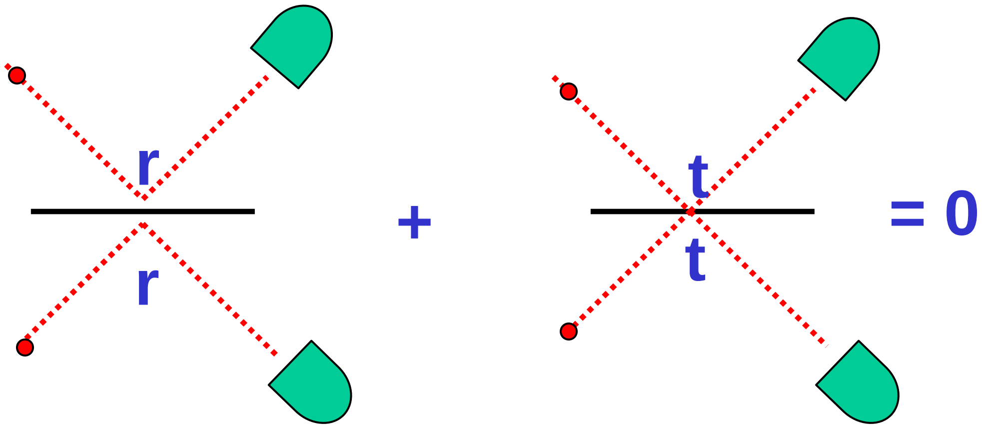
- For each input state we measure H-H, H-V, V-45, 45-LHS, etc. coincidence rates



# Our Black Box

The (not-so) simple 50/50 beamsplitter

Codename:
Bell-state Filter

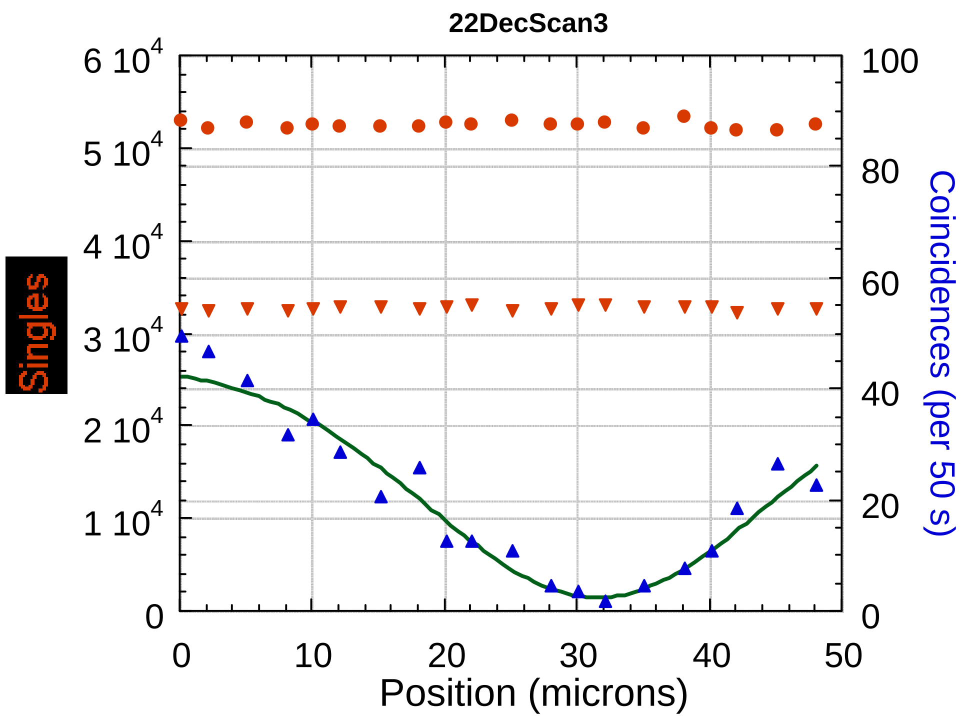



| **Bell - State** | **Coincidence Counts** |
|---|---|
| $|\Phi^{+}> = |HH> + |VV>$ | **No (symmetric)** |
| $|\Phi^{-}> = |HH> - |VV>$ | **No (symmetric)** |
| $|\Psi^{+}> = |HV> + |VH>$ | **No (symmetric)** |
| $|\Psi^{-}> = |HV> - |VH>$ | **Yes! (anti-symmetric)** |

Uses: Quantum Teleportation, Quantum Repeaters, CNOT

**Our Goal: use process tomography to test this filter.**

# Hong-Ou-Mandel Interference




> 85% visibility for HH and VV polarizations

HOM acts as a filter for the Bell state:

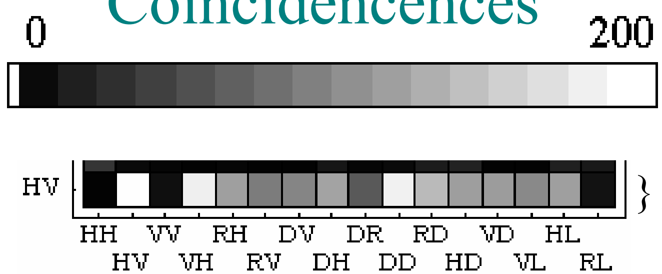
$\Psi^{-} = (HV\text{-}VH)/\sqrt{2}$

Goal: Use Quantum Process Tomography to find the *superoperator* which takes $\rho_{in} \rightarrow \rho_{out}$

# Measuring the Superoperator

1. Input: $|\Psi\rangle = |H\rangle\,|V\rangle$

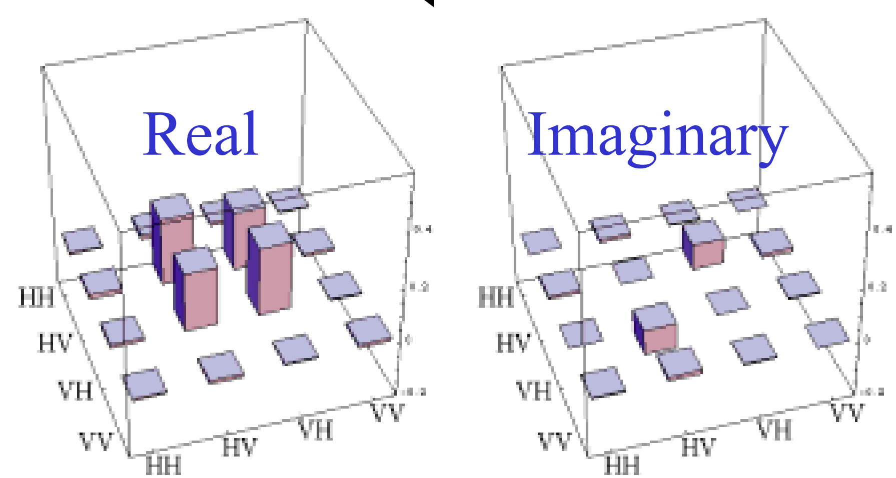
2. Measurement




16 analyzer settings

3. Output:

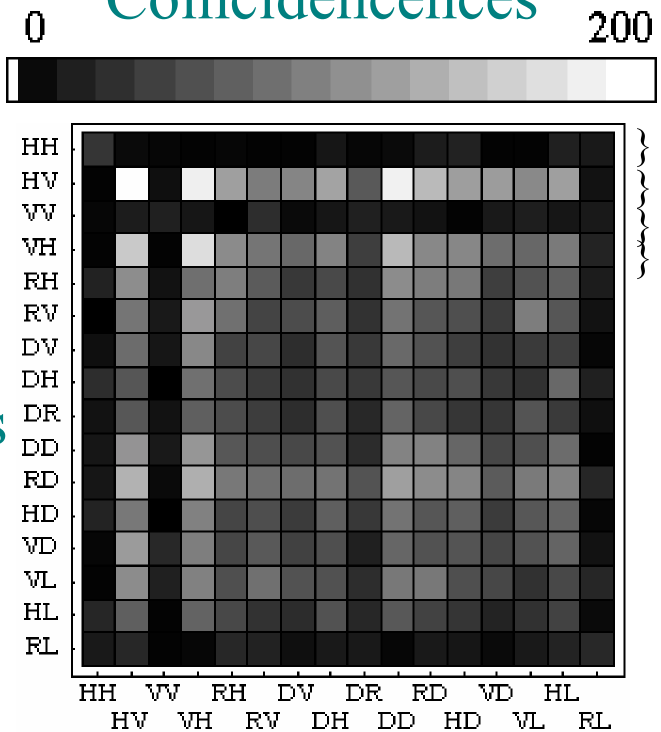
$\rho_{out} =$



# Repeat for 16 Input States

Coincidencences

16 input states

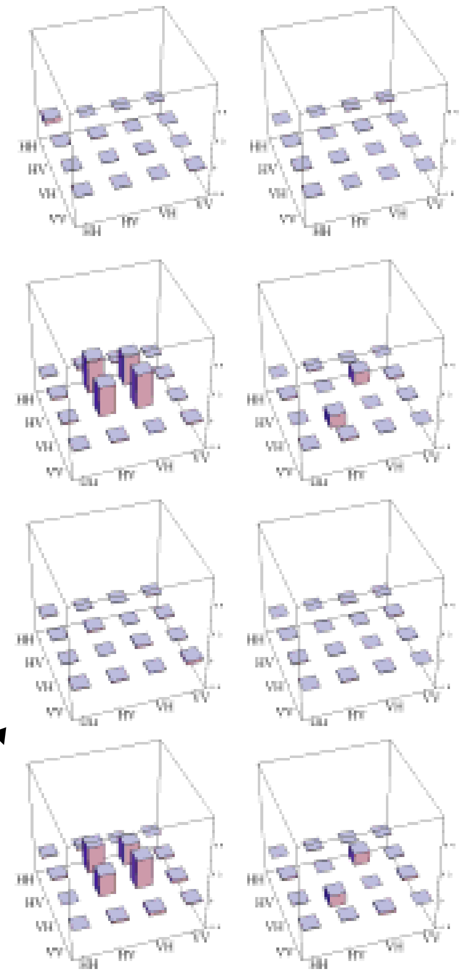



16 analyzer settings

etc.

Output ρ

Input



HH

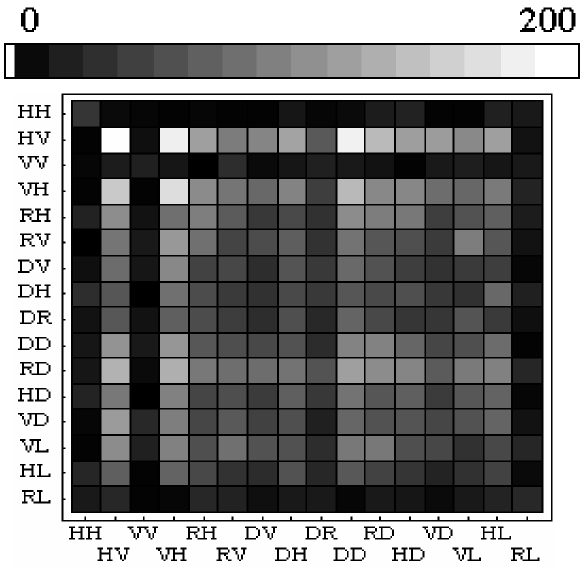
HV

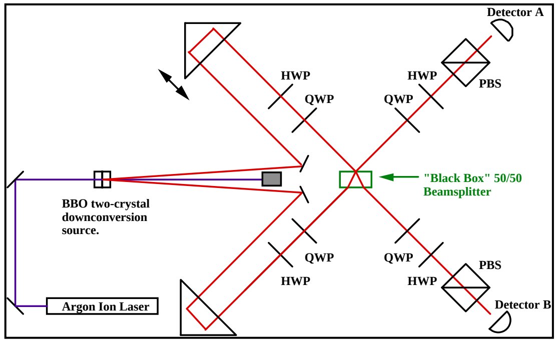
VV

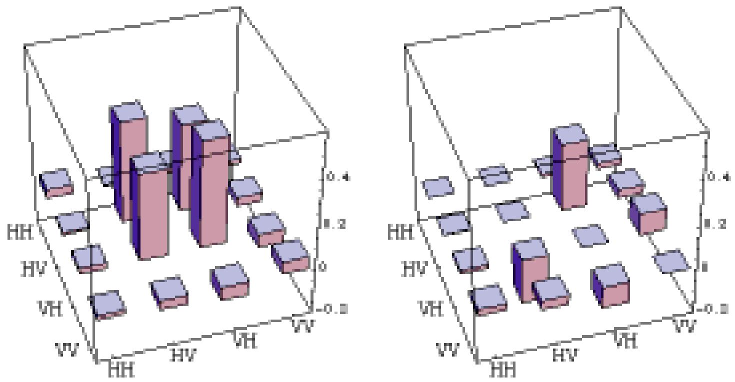
VH

# Testing the Superoperator

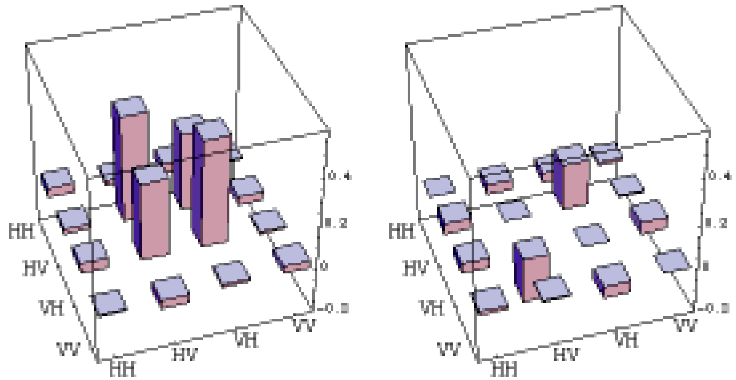
**LL** = input state




Predicted $N_{photons} = 297 \pm 14$

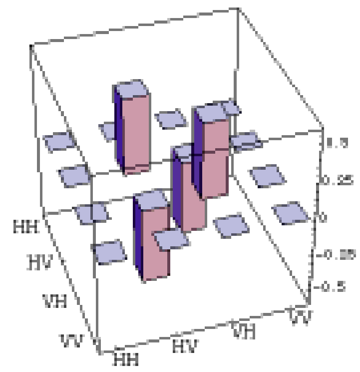


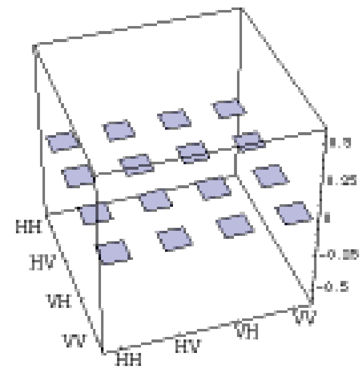

Observed $N_{photons} = 314$

# So, How's Our Bell-State Filter?

In: Bell singlet state: $\Psi^{-} = (HV\text{-}VH)/\sqrt{2}$

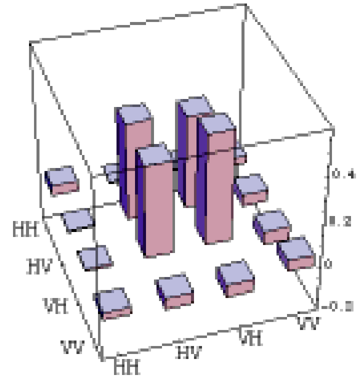
$$\Psi^{-} = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix} =$$

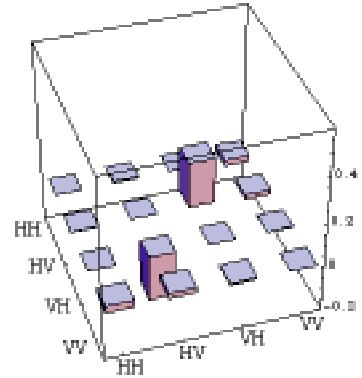



Out: $\neq \Psi^{-}$, but is a different maximally entangled state:

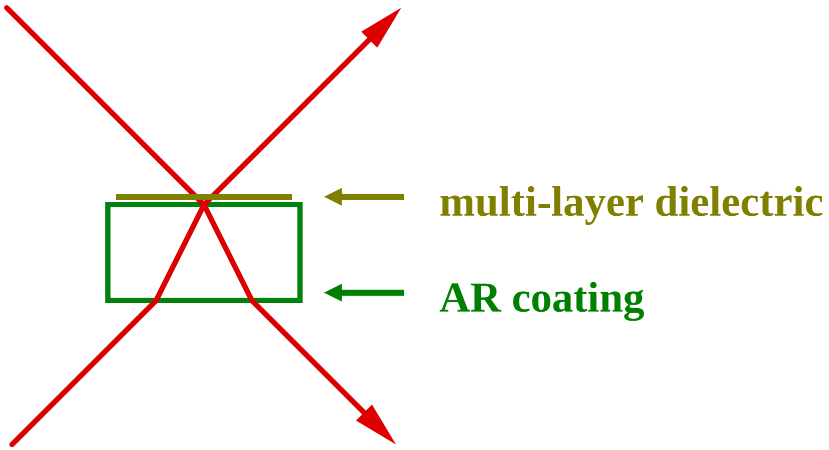
$$\Psi^{-} \neq$$

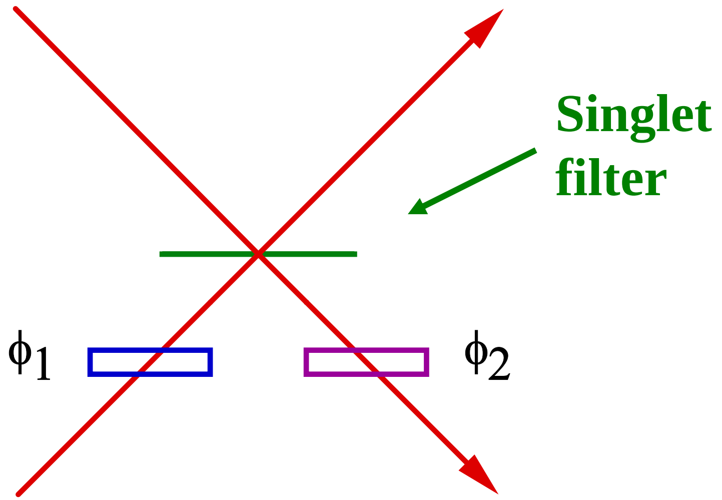


# Model of real-world beamsplitter






**45° "unpolarized" 50/50 dielectric beamsplitter at 702 nm (CVI Laser)**

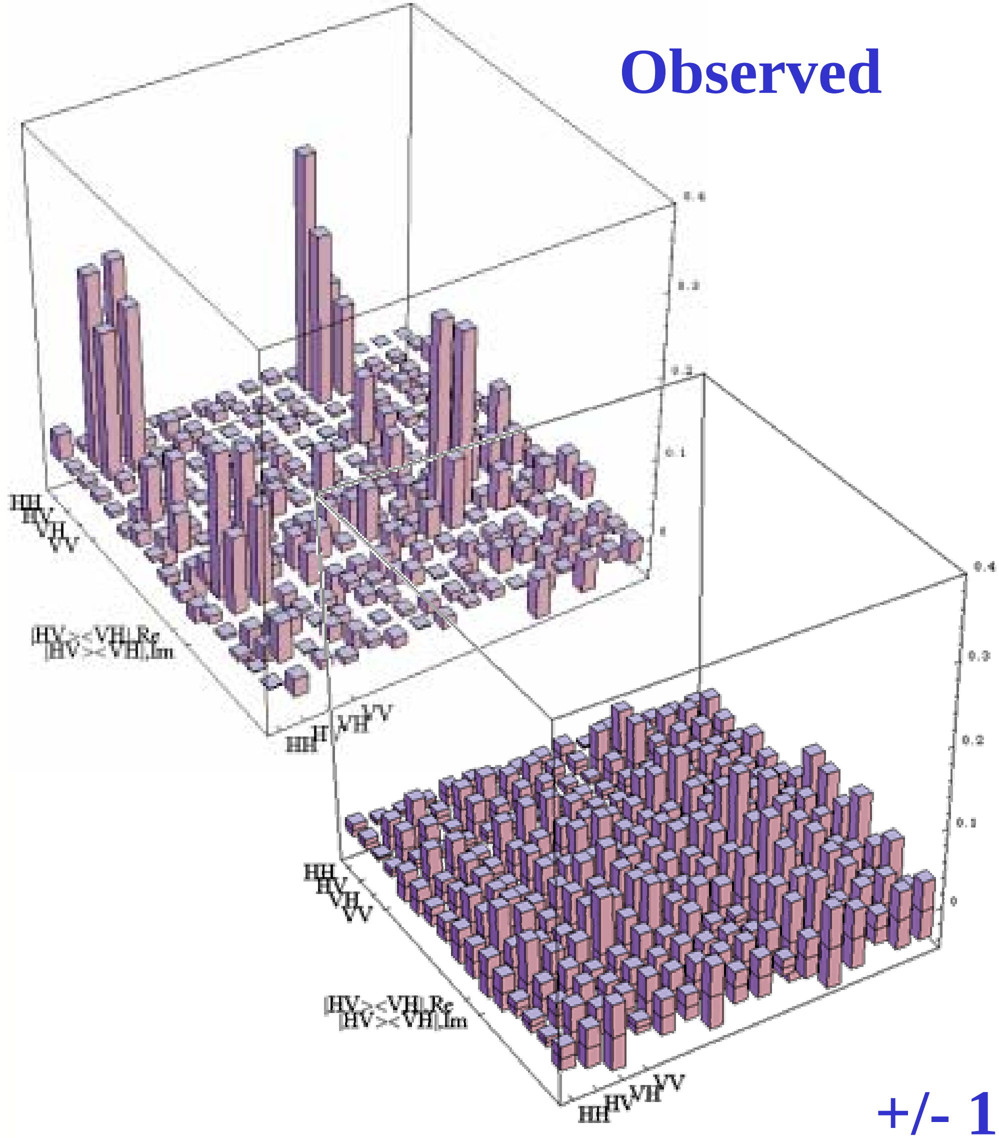
**birefringent element**
**+**
**singlet-state filter**
**+**
**birefringent element**

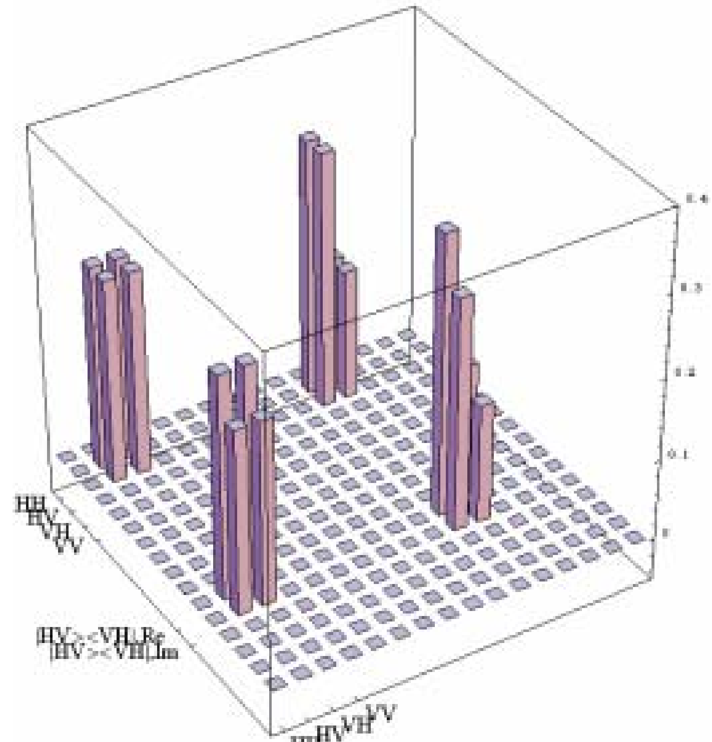
**Best Fit:** $\phi_1 = 0.76\,\pi$
$\phi_2 = 0.80\,\pi$

# Comparison to Measured Superop.



Observed




Predicted

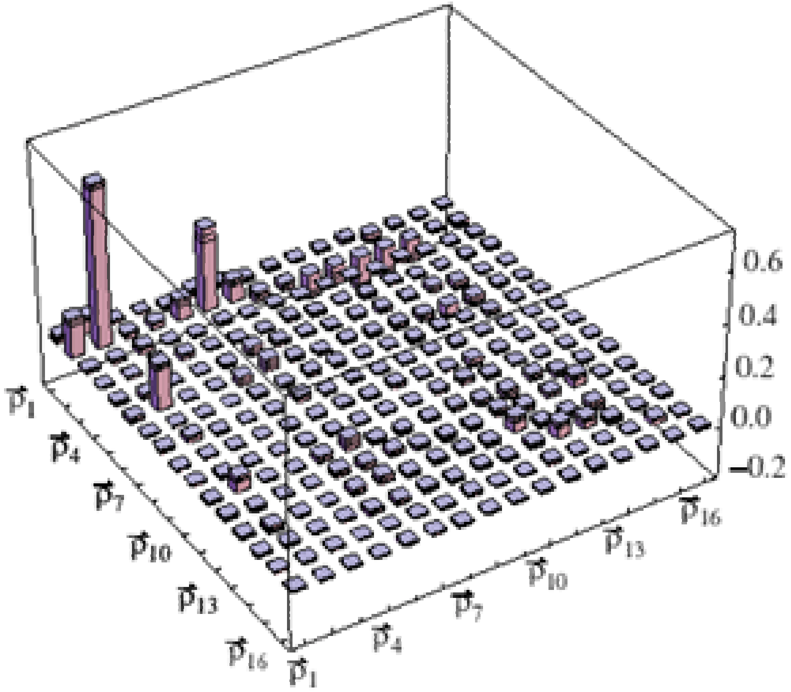
$$\chi^2 = 477$$
$$\text{N-d.f.} = 253$$

+/- 1σ (statistical)

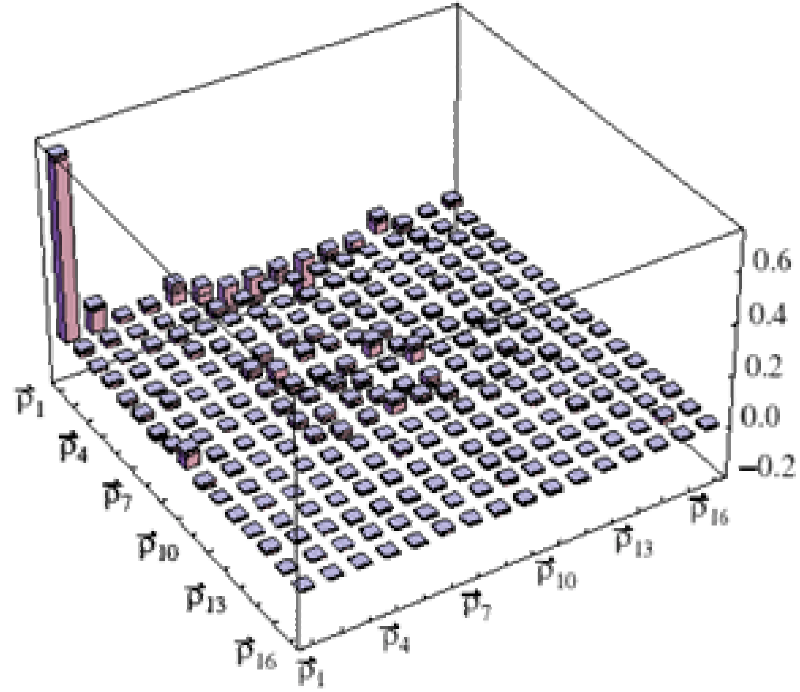
# Comparison to Ideal Filter



**Measured superoperator, in Bell-state basis:**




**Superoperator after transformation to correct polarisation rotations:**




**A singlet-state filter would have a single peak, indicating the one transmitted state.**

**Dominated by a single peak; residuals allow us to estimate degree of decoherence and other errors.**

# Process Tomography with Atoms



- State reconstruction is performed on a system of cold atoms in an optical lattice.

- Using various input states Quantum process tomography on time dependent sinusoidal potentials is performed. This results in a superoperator which completely characterizes the evolution of a state in the potential.

$\rho_{in}$ $\longrightarrow$  $\longrightarrow$ $\rho_{out}$

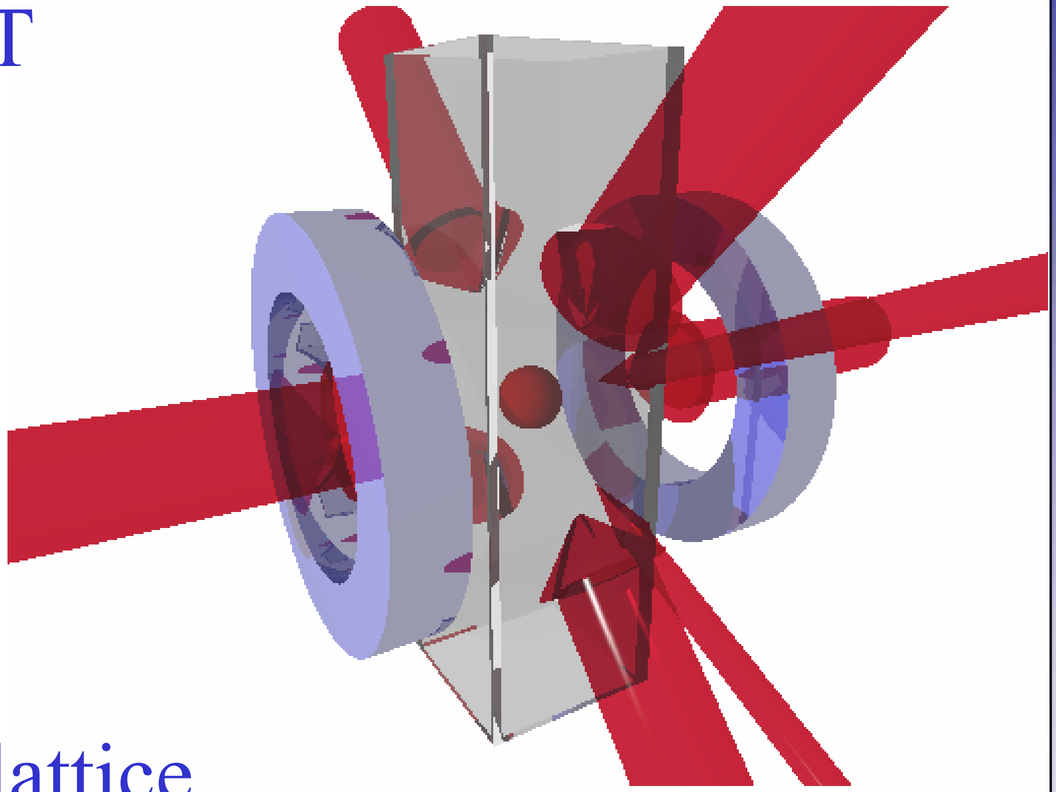
"Black Box"

$$\rho_{out} = \hat{\varepsilon}(\rho_{in})$$

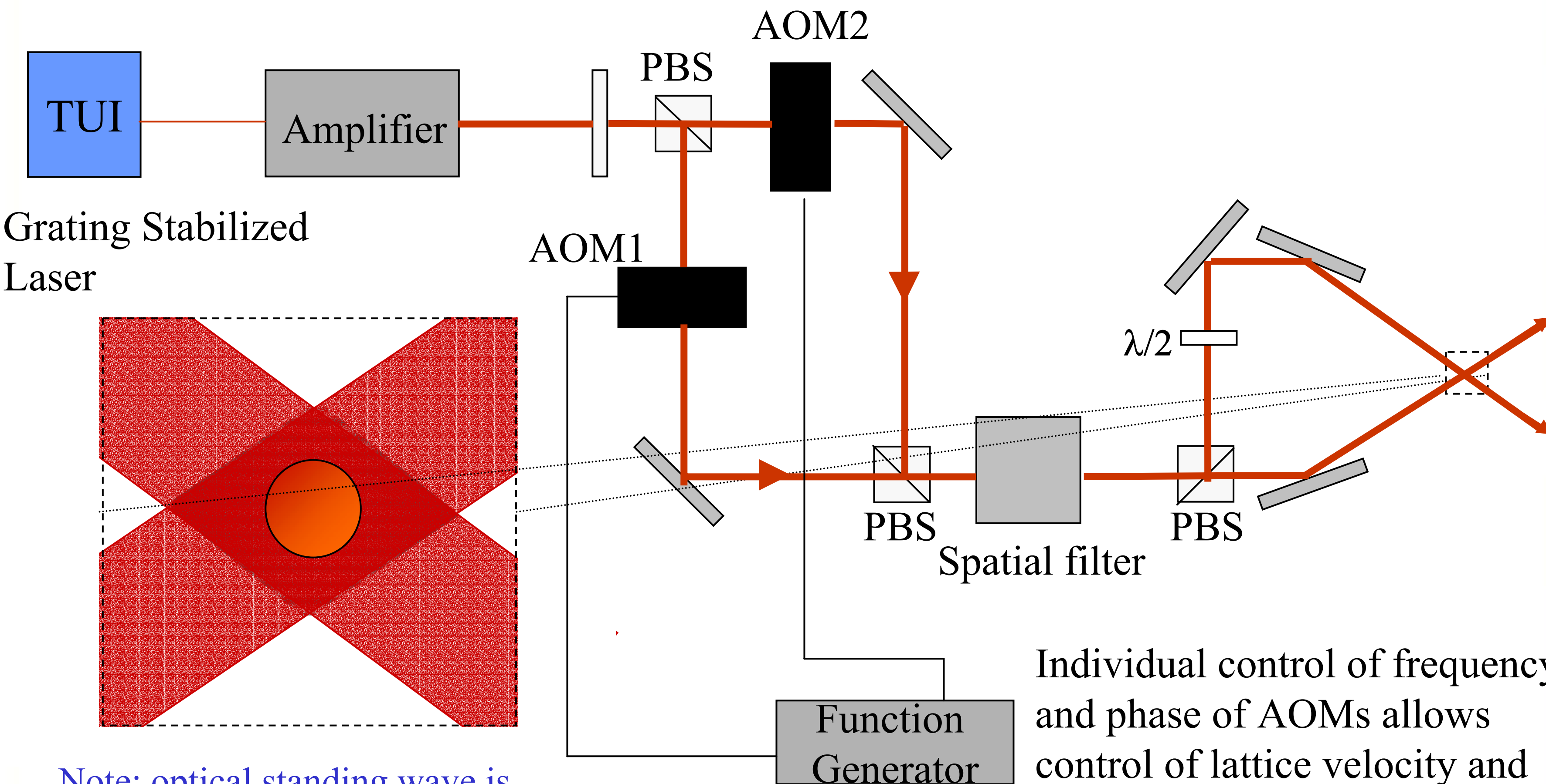
$\hat{\varepsilon} \equiv$ superoperator of system

# The System

- Rb vapour cell MOT
- $10^8$ atoms
- Cooled to 6 μK
- Load a 1-D optical lattice during molasses stage

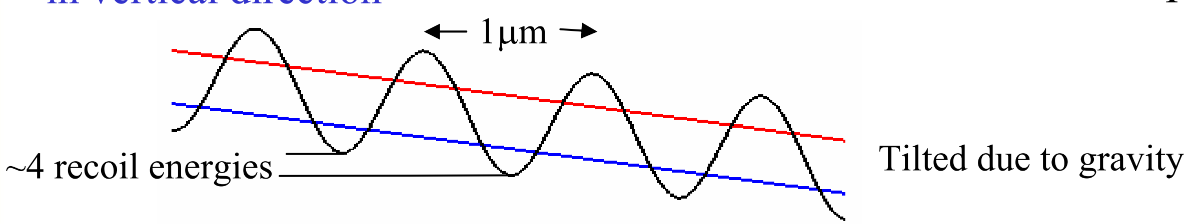


# Experimental Setup




Individual control of frequency and phase of AOMs allows control of lattice velocity and position.

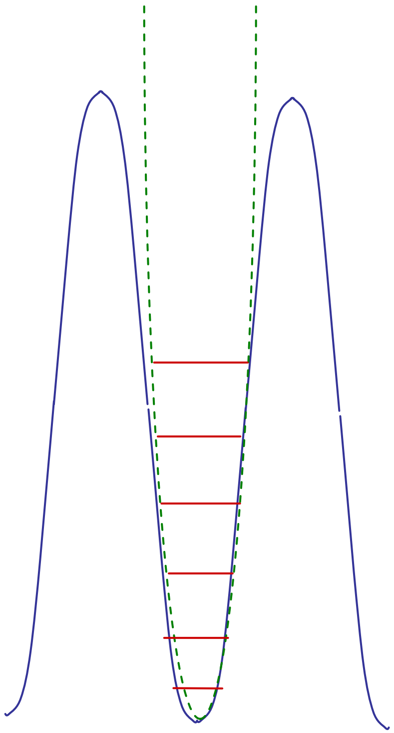
Note: optical standing wave is in vertical direction



# Optical Lattices

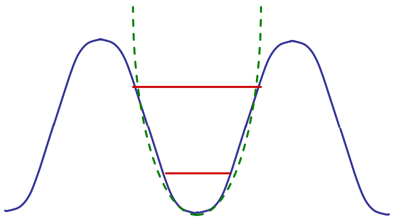


Interference of lasers creates a standing wave. Atoms experience an energy shift proportional to the light intensity. Creates a sinousoidally varying potential.



Individual lattice wells can be thought of as harmonic oscillators.

More accurate for deeper lattices, but still a valid approximation for 2 state lattices



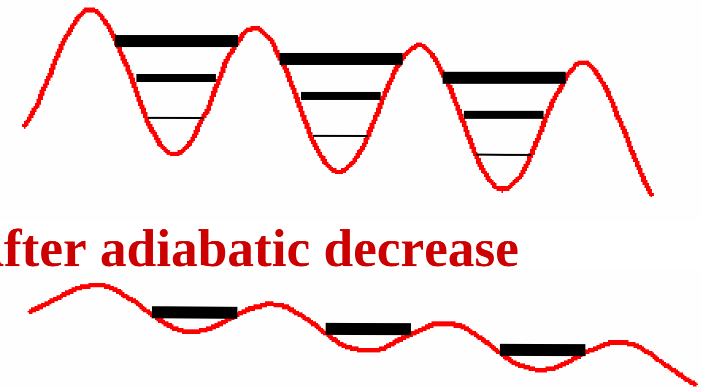
$$\begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{12}^{*} & 1-\rho_{11} \end{bmatrix}$$

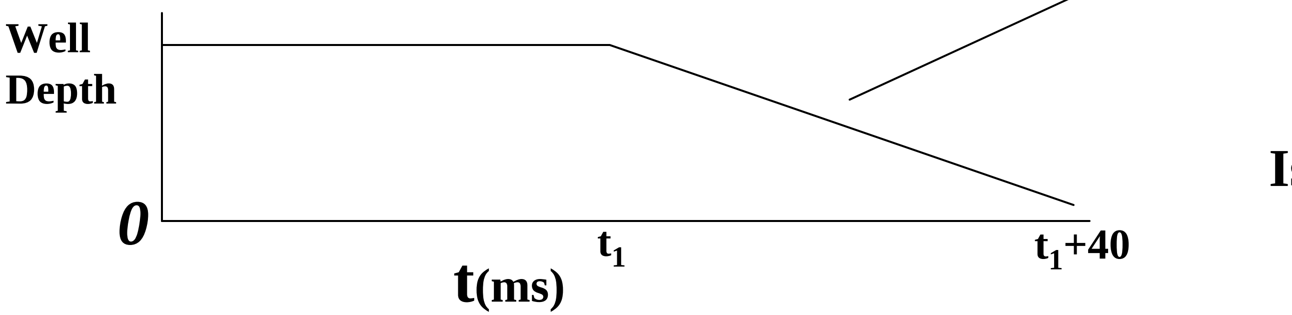
In our experiments, we measure the motional density operator of the atoms in our lattice

*energy spacings
*energy state populations
*coherences

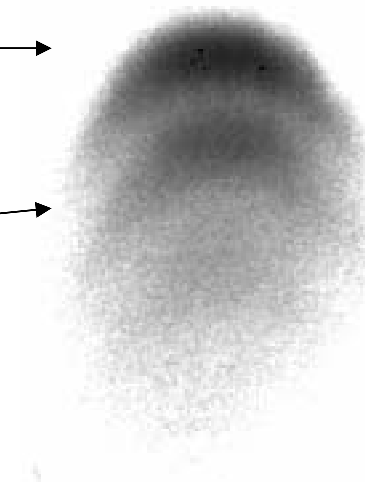
# Measuring state populations

## Initial Lattice

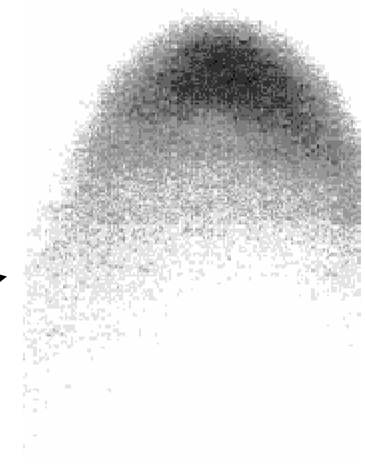


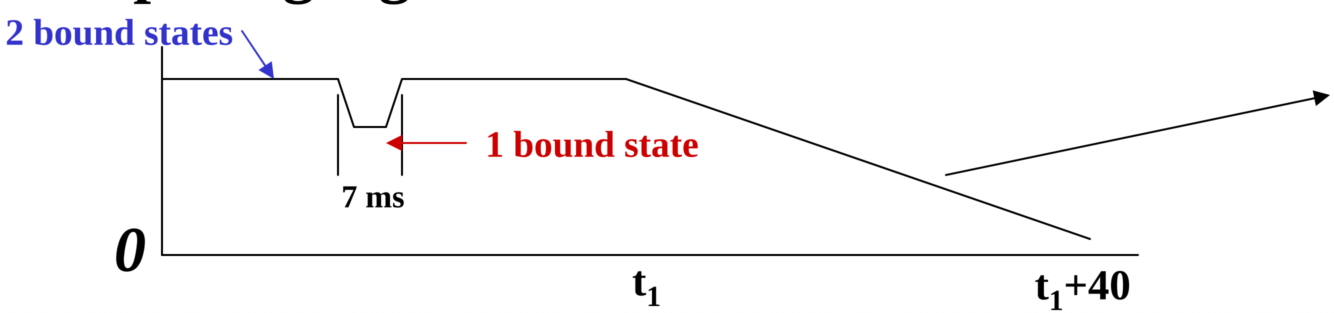

Thermal state




Isolated ground state

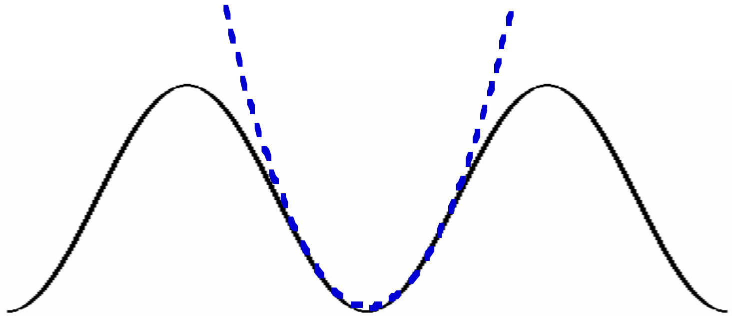


## Preparing a ground state



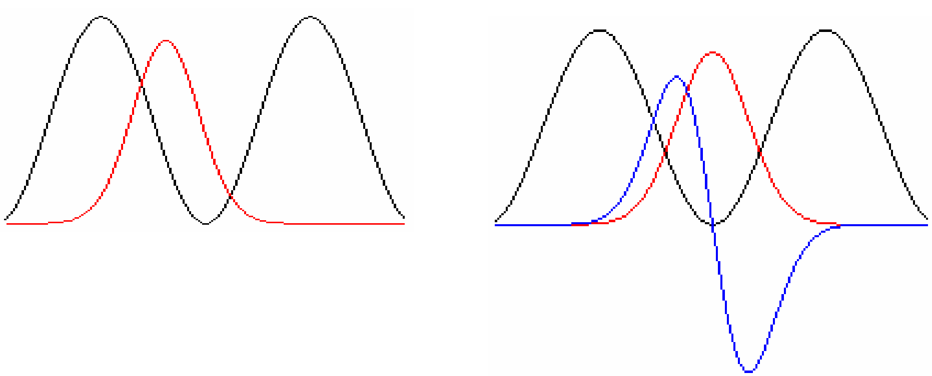
# Displacements and Rotations

To apply rotation and displacement operators, $R(\omega t)$ and $D(\varepsilon)$ first wait a time, $t$ to let the state rotate through $\theta = \omega t$



$$U(x) = \frac{1}{2} m\omega^2 x^2$$

Next, perform the displacement, $D(\varepsilon)$



$$D|0\rangle \approx a|0\rangle + b|1\rangle + \textit{unbound atoms}$$

$$\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} \rightarrow \begin{bmatrix} a & b \\ -b^* & a \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

*Measured experimentally*

Instead of displacing the state, we displace the trap by changing the phase of one lattice beam.

$\Phi = 45^o$ → 125nm

# Measuring 2 State Systems



For a 2 state system, state reconstruction can be performed with 3 measurements by projecting the unknown state, $\rho$, onto the set of known states, $|\Phi(\varepsilon,\omega t)\rangle$ :

$$m(\varepsilon,\omega t) = (a\langle 0| + e^{-i\omega t} b\langle 1|)\,\rho\,(a|0\rangle + e^{+i\omega t} b|1\rangle)$$

$$= \langle 0|D^{\dagger}(\varepsilon)R(\omega t)\rho R^{\dagger}(\omega t)D(\varepsilon)|0\rangle = \langle \Phi(\varepsilon,\omega t)|\rho|\Phi(\varepsilon,\omega t)\rangle$$

$D(\varepsilon)$ is the displacement operator
$R(\omega t)$ is the rotation operator
Acting on the vacuum state, they create a coherent mix of the ground and excited state:

$$|\Phi(\varepsilon,\omega t)\rangle = a|0\rangle + e^{+i\omega t} b|1\rangle = R^{\dagger}(\omega t)D(\varepsilon)|0\rangle$$

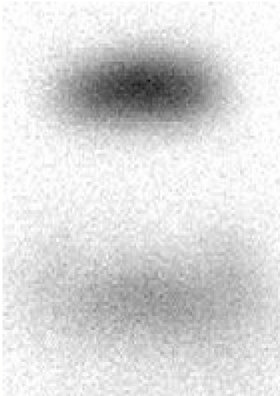
# Measuring 2 State Systems cont'd

**E.g., state tomography on** $0.8|0\rangle + i\,0.6|1\rangle$

$|\Phi(\varepsilon,\omega t)\rangle = |0\rangle$

$|\Phi(\varepsilon,\omega t)\rangle = 0.9|0\rangle + 0.4|1\rangle$

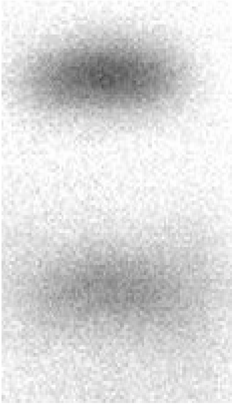
$|\Phi(\varepsilon,\omega t)\rangle = 0.9|0\rangle + i0.4|1\rangle$



$\rho_{11}$

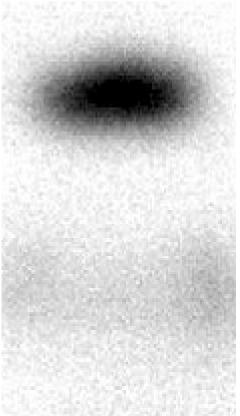
$\rho_{22}$

$\varepsilon = 0$



$\rightarrow \rho_{12} + \rho_{12}^*$

$\varepsilon = 125nm$
$\omega t = 0$



$\rightarrow \rho_{12} - \rho_{12}^*$

$\varepsilon = 125nm$
$\omega t = \pi/2$

$$\Re e(\rho_{12}) = \tfrac{1}{2}(\rho_{12} + \rho_{12}^*) = (m(\varepsilon,\omega t = 0) - a^2\rho_{11} - b^2\rho_{22})/(2ab)$$

$$\Im m(\rho_{12}) = \tfrac{1}{2}(\rho_{12} - \rho_{12}^*) = (m(\varepsilon,\omega t = \pi/2) - a^2\rho_{11} - b^2\rho_{22})/(2ab)$$

# QPT of Decoherence

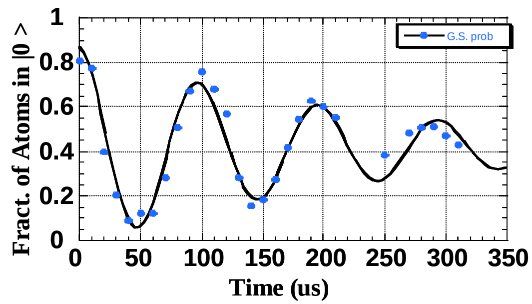


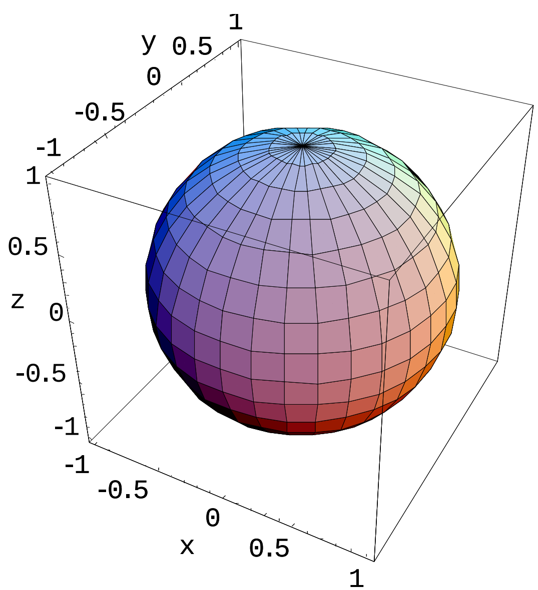
$$\rho_{in}: \begin{bmatrix} .9 & -.01 + .02\,I \\ -.01 - .02\,I & .1 \end{bmatrix} \quad \begin{bmatrix} .60 & .01 + .02\,I \\ .01 - .02\,I & .40 \end{bmatrix} \quad \begin{bmatrix} .69 & .41 + .010\,I \\ .41 - .010\,I & .31 \end{bmatrix} \quad \begin{bmatrix} .69 & .010 - .35\,I \\ .010 + .35\,I & .31 \end{bmatrix}$$

Process:
Sitting in the lattice
for 1 period.

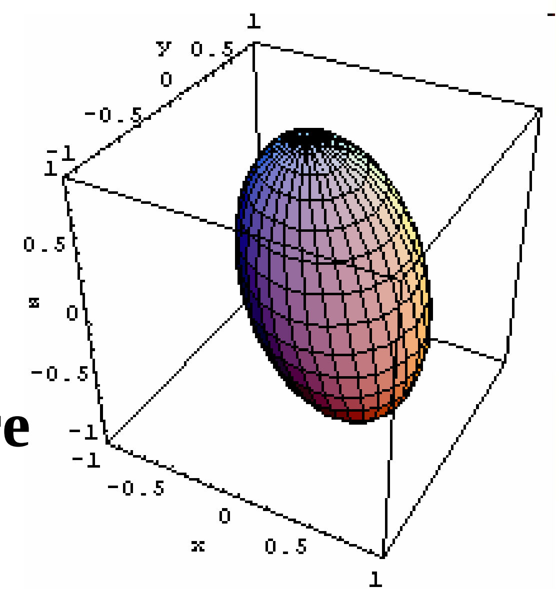
$\rho_{in} \rightarrow \rho_{out}$

$$\rho_{out}: \begin{bmatrix} .9 & -.01 \\ -.01 & .1 \end{bmatrix} \quad \begin{bmatrix} .60 & .05-.02I \\ .05+.02I & .40 \end{bmatrix} \quad \begin{bmatrix} .69 & .28-.010I \\ .28+.010I & .31 \end{bmatrix} \quad \begin{bmatrix} .69 & 0. - .26\,I \\ 0. + .26\,I & .31 \end{bmatrix}$$



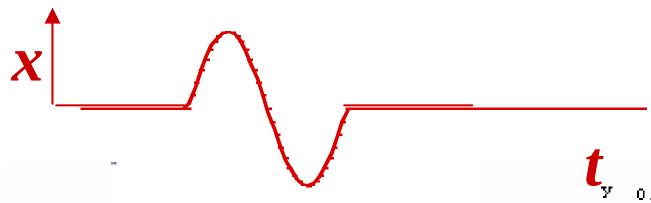



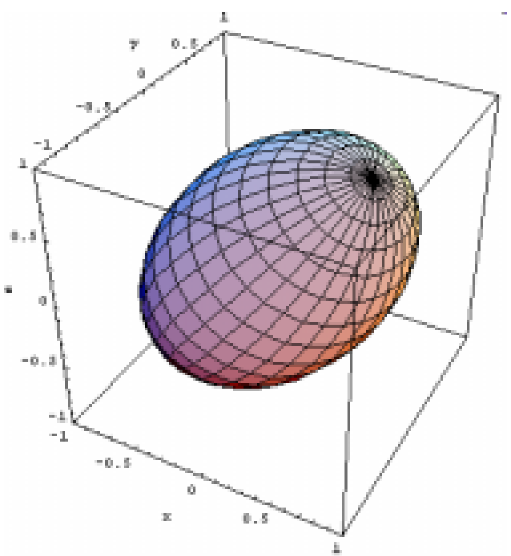

**Initial Bloch Sphere** → **Final Bloch Sphere**

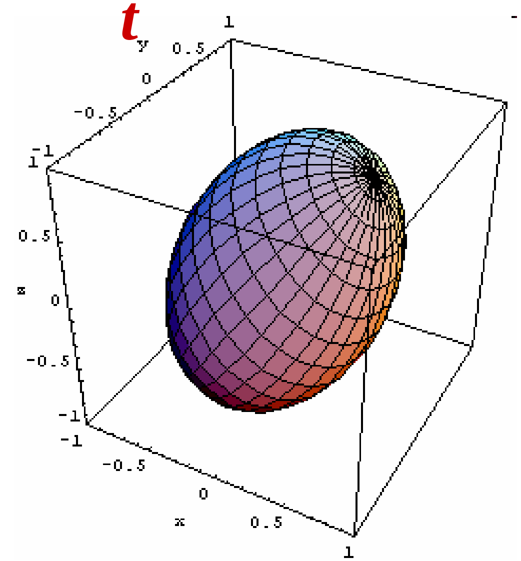


# QPT of Driving Oscillations

Operation: Resonantly shake the lattice.



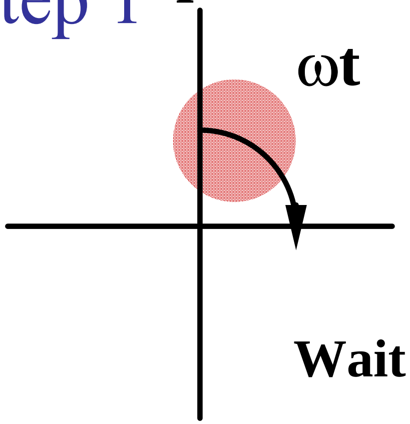


Observed Bloch Sphere

Modelled Bloch Sphere from theory (Harmonic oscillator plus decoherence from previous measurement)

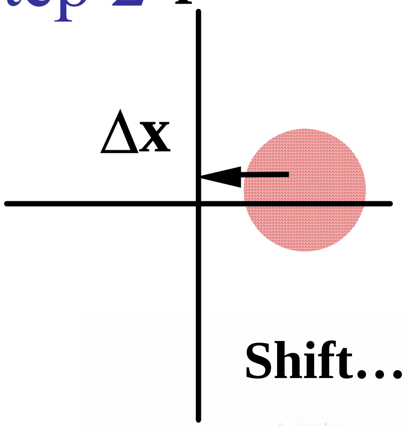
# Quantum State Reconstruction

Step 1



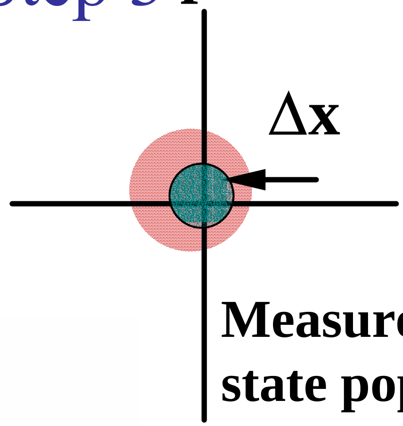

**Wait…**

Step 2




**Shift…**

Step 3

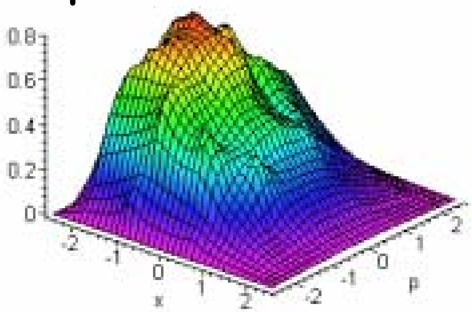



**Measure ground state population**

**Husimi Representation:**

$$Q(\alpha) = \frac{1}{\pi}\langle\alpha|\rho|\alpha\rangle$$

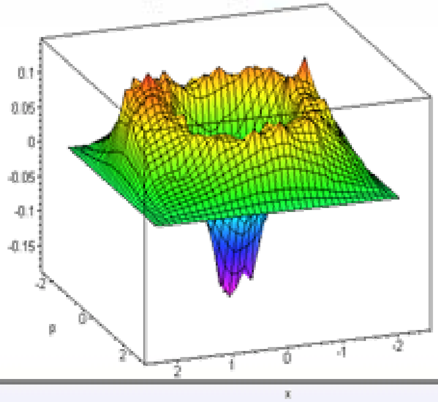
$$Q_{H.O.}(0,0) = P_g$$



← **Coherent state**

**Wigner Representation:**

$$W_{H.O.}(0,0) = \frac{1}{\pi}\sum_{n=0}^{\infty}(-1)^n P_n$$



← **First two terms in the Wigner Function**

# Husimi Dist. of an Inverted State



**Creating a mixed inverted state**

Step 1: Resonantly oscillate position of lattice for 3 oscillations

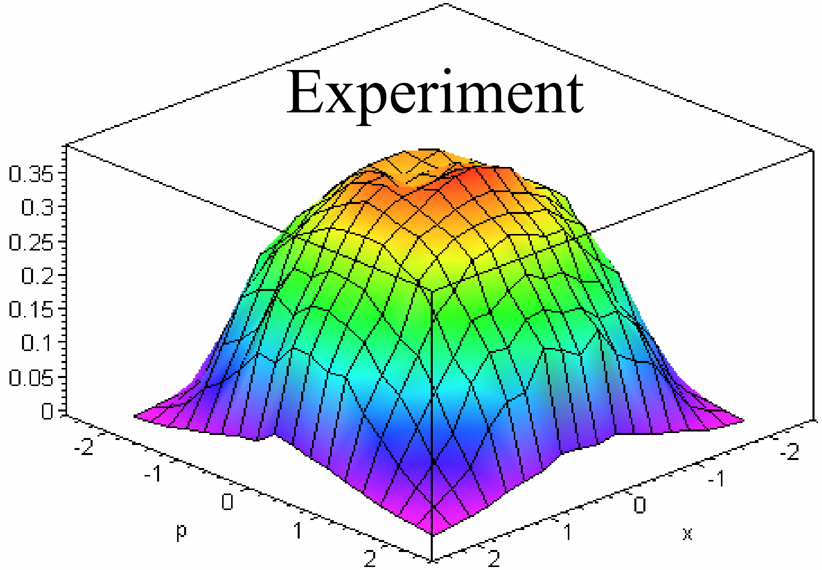
- generates state with $P_e = 2P_g$

Step 2: Decrease well depth until 2 states are bound

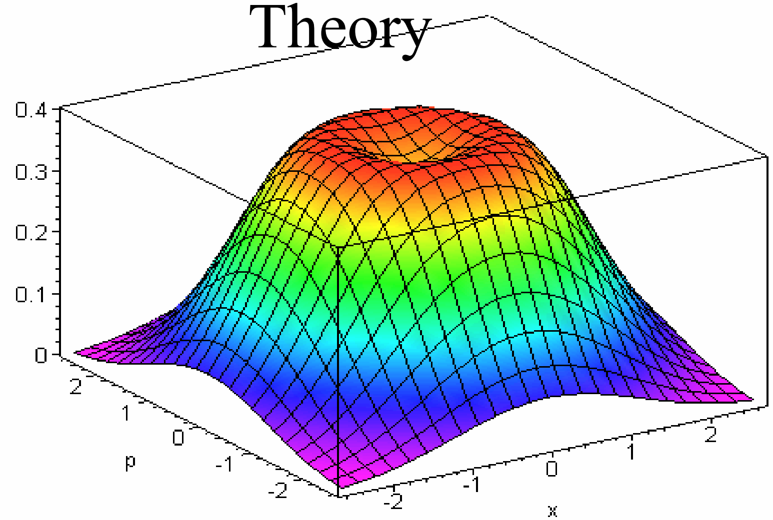
Step 3: Add a 3ms delay before measurements.

- sample decoheres (dephasing?)

Experiment



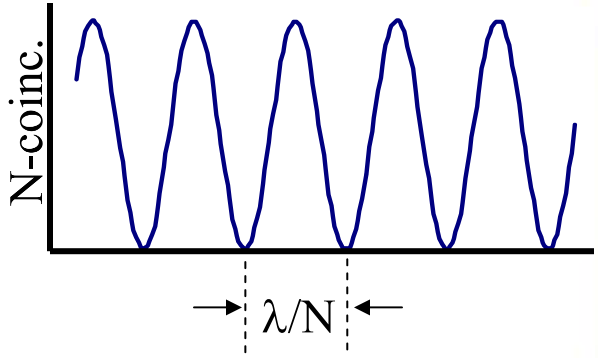

Theory



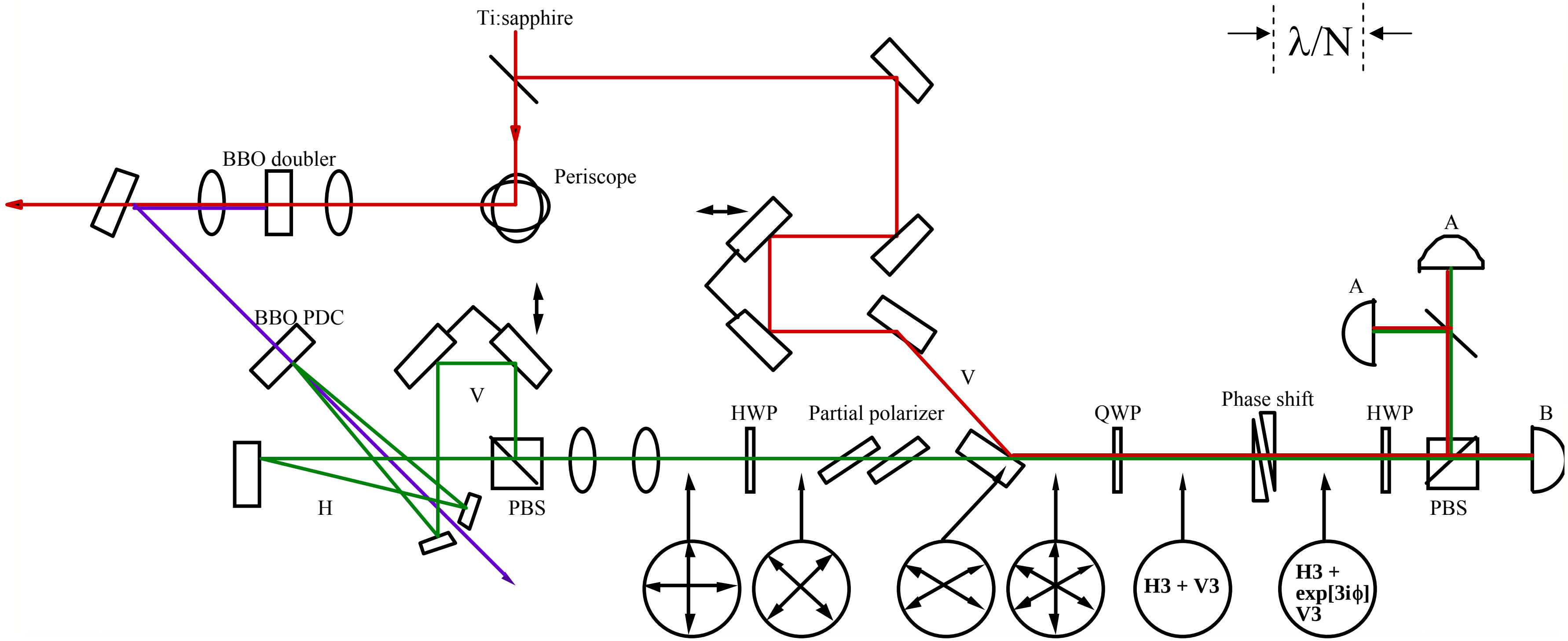
# Current Work: High-Noon States

- We can "mash" together N photons with polarizations distributed evenly across the Bloch sphere to create $|N_H 0_V\rangle + |0_H N_V\rangle$

→ N times more interferometric precision




## 3-Noon Experimental Setup



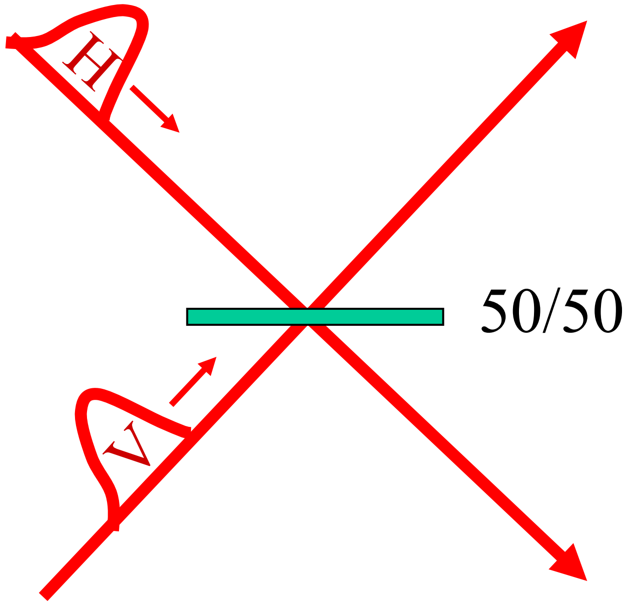
# Current Work: Resource Optimal DFS Identification

**Our process:**

Prob =1/2:  Swap  ie. $|HV\rangle \rightarrow |VH\rangle$

Prob =1/2:  Identity ie. $|HV\rangle \rightarrow |HV\rangle$

Decoherence!




**The Goal:  Find a method that determines which states form a decoherence-free subspace (DFS) without knowing the decoherence mechanism or doing full QPT.0**

# Current Work: Tailored Error Correction for Optical lattices

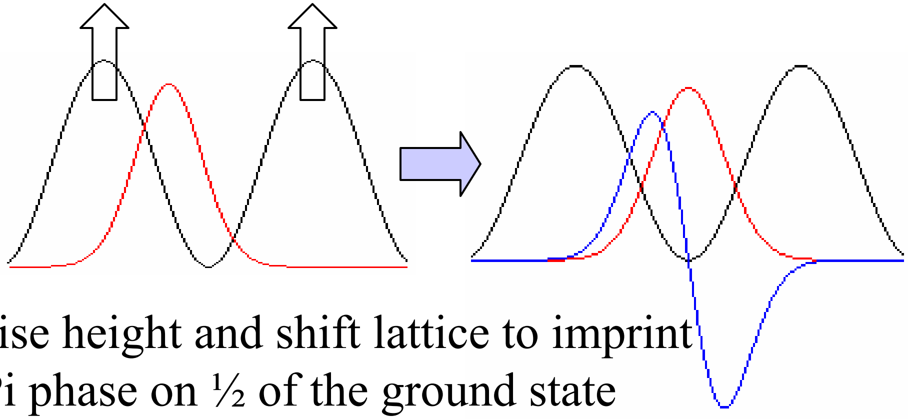
## The Goal: Undo dephasing of states in the lattice

**Stage 1: Phase Imprinting**

Apply Pi pulses to reverse dephasing

Fast = Bang Bang QEC

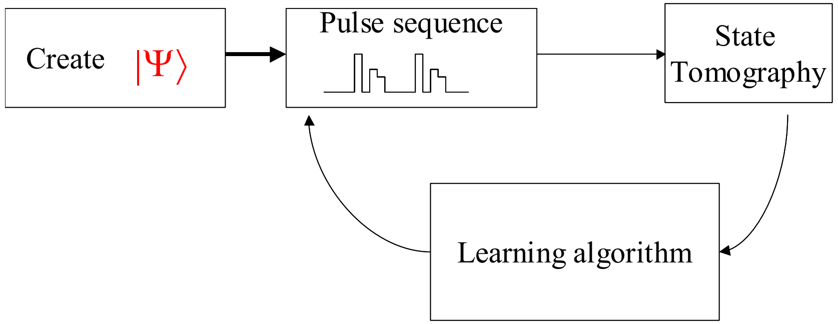
Slow = Spin Echo



Raise height and shift lattice to imprint a Pi phase on ½ of the ground state

**Stage 2: Learning Algorithm**

- Measure state
- Use fidelity as a cost function
- Feedback to the lattice through its phase, intensity and velocity



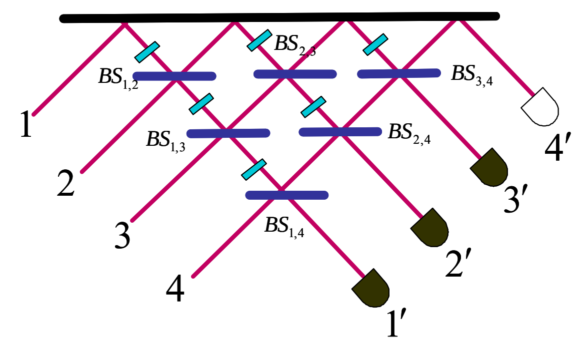
# Current Work: Nonorthogonal State Discrimination



**Projective measurements can distinguish these three non-orthogonal states at most 1/3 of the time:**

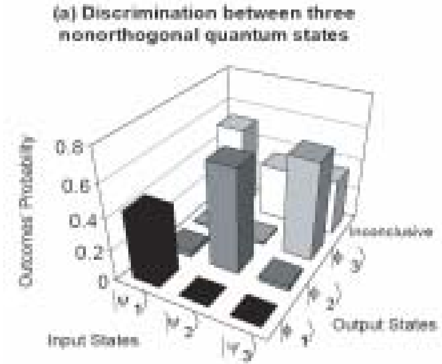
$$|\psi_1\rangle_{in} = \begin{pmatrix} \sqrt{2/3} \\ 0 \\ 1/\sqrt{3} \end{pmatrix} \; ; \; |\psi_2\rangle_{in} = \begin{pmatrix} 0 \\ 1/\sqrt{3} \\ \sqrt{2/3} \end{pmatrix} \; ; \; |\psi_3\rangle_{in} = \begin{pmatrix} 0 \\ -1/\sqrt{3} \\ \sqrt{2/3} \end{pmatrix}$$

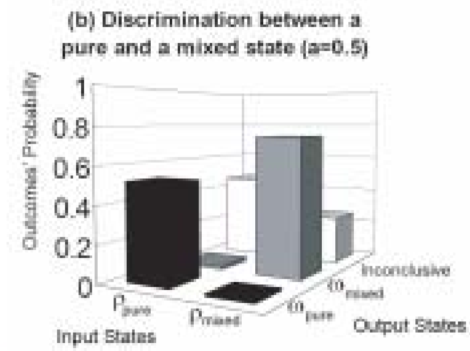
**But a unitary transformation in a 4D space produces:**




$$|\psi_1\rangle_{out} = \begin{pmatrix} 1/\sqrt{3} \\ 0 \\ 0 \\ \sqrt{2/3} \end{pmatrix} \quad |\psi_2\rangle_{out} = \begin{pmatrix} 0 \\ 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} \quad |\psi_3\rangle_{out} = \begin{pmatrix} 0 \\ -1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}$$

**… these states can be distinguished 55% of the time**



# Summary

## PHOTONS

- Quantum process tomography for two polarized photons
- Superoperator for a not so perfect Bell-state filter

## ATOMS

- State Tomography in a 2 bound-state lattice: Coherent state, inverted state, Fock State
- Quantum process tomography: Superoperator for “natural” decoherence and single qubit rotations

## CURRENT WORK

- Tailored error correction for decohering swap operation on photons (Optimal QPT measurements)
- Tailored pulse sequences to investigate and undo decoherence in the optical lattices (Learning Algorithm)