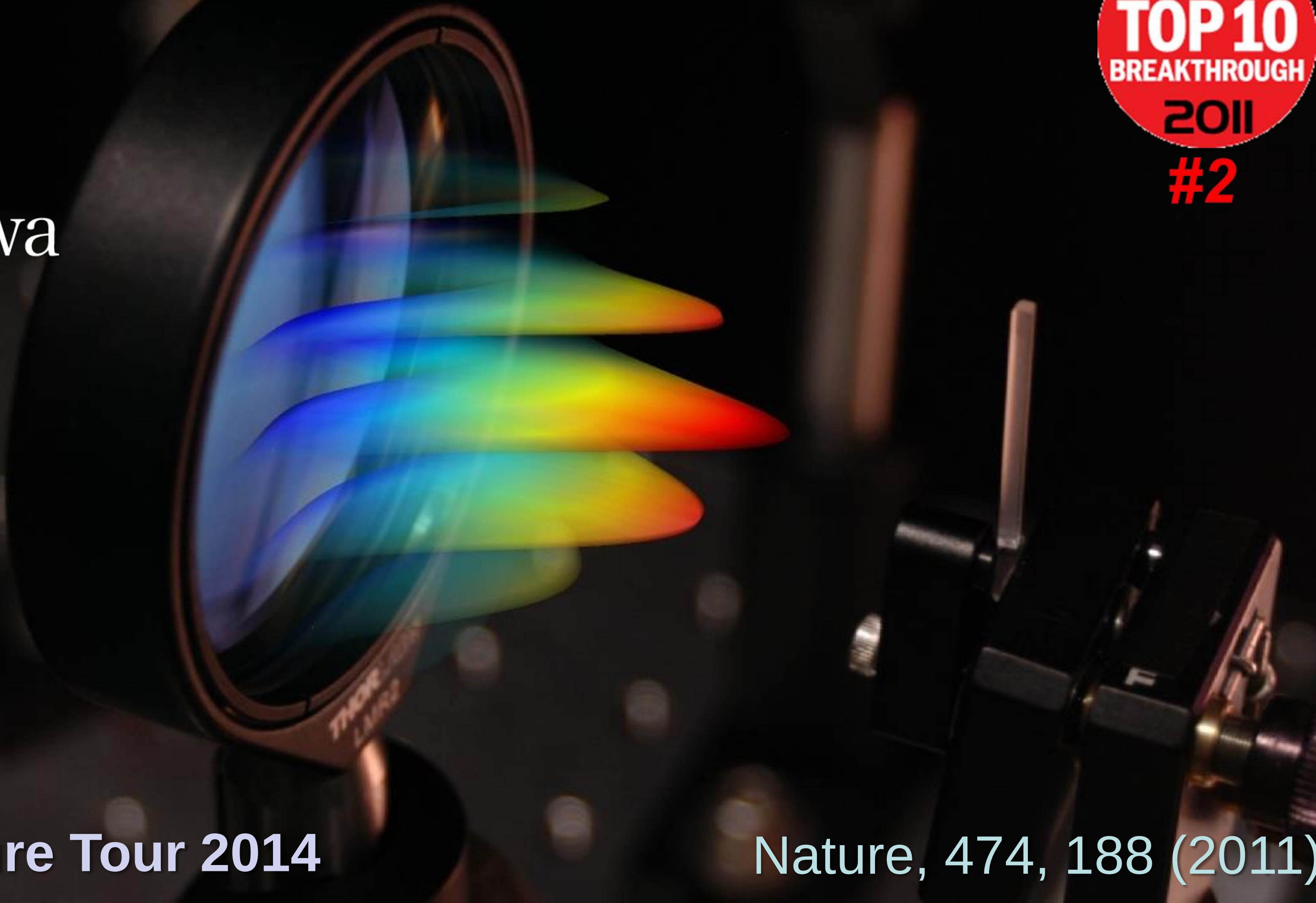
Seeing is Believing
Direct Observation of the Wavefunction

Jeff Lundeen

uOttawa

physicsworld
TOP 10
BREAKTHROUGH
2011
#2



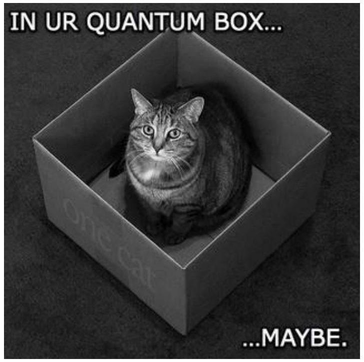
CAP Lecture Tour 2014

Nature, 474, 188 (2011)

Outline

1. The wavefunction revisited and reviewed.

2. Progress in understanding the wavefunction

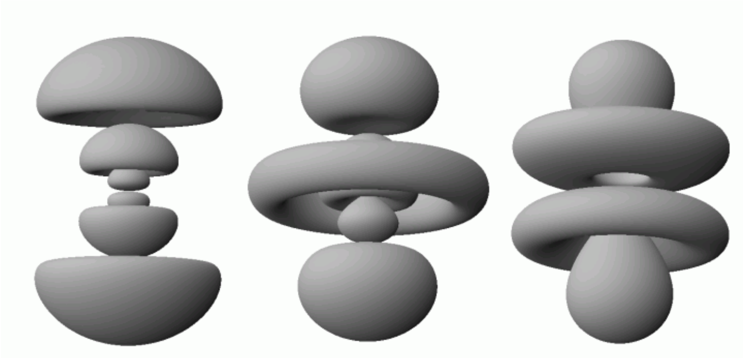


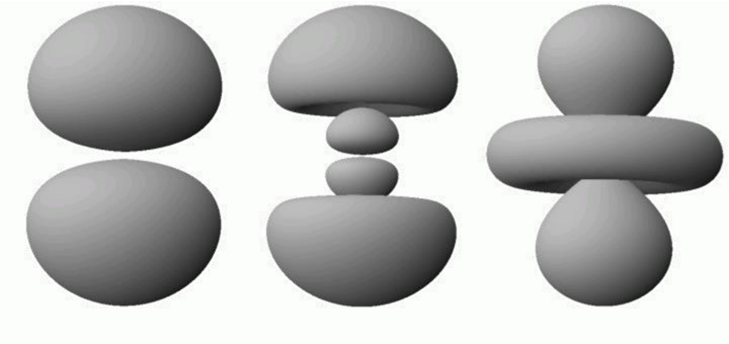
3. How we directly measure the wavefunction.

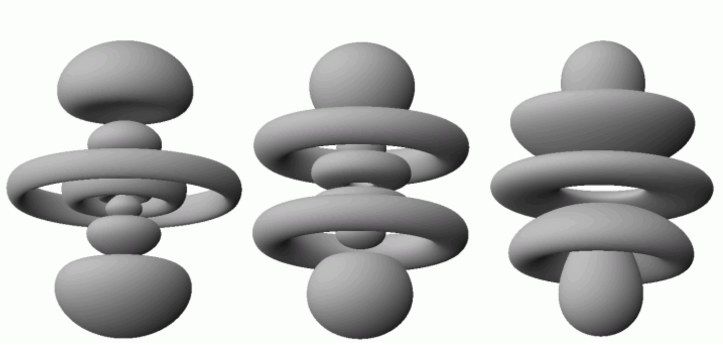
The Wavefunction $\Psi(r,t)$

- In classical physics, a particle has a single position r and momentum p.
- The Heisenberg uncertainty principle $\Delta x \, \Delta p \geq \hbar/2$ implies that this is not the case for a quantum particle.
- In quantum physics, a particle is associated a distribution of positions and momenta – the wavefunction, $\boldsymbol{\Psi(r)}$.

e.g. the Hydrogen Electron Orbitals

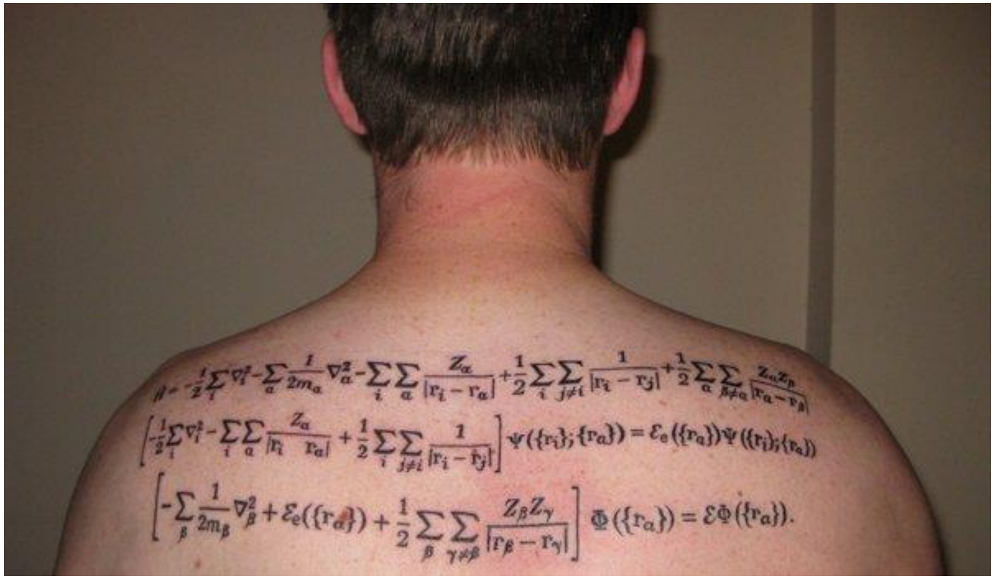






The Schrödinger Equation

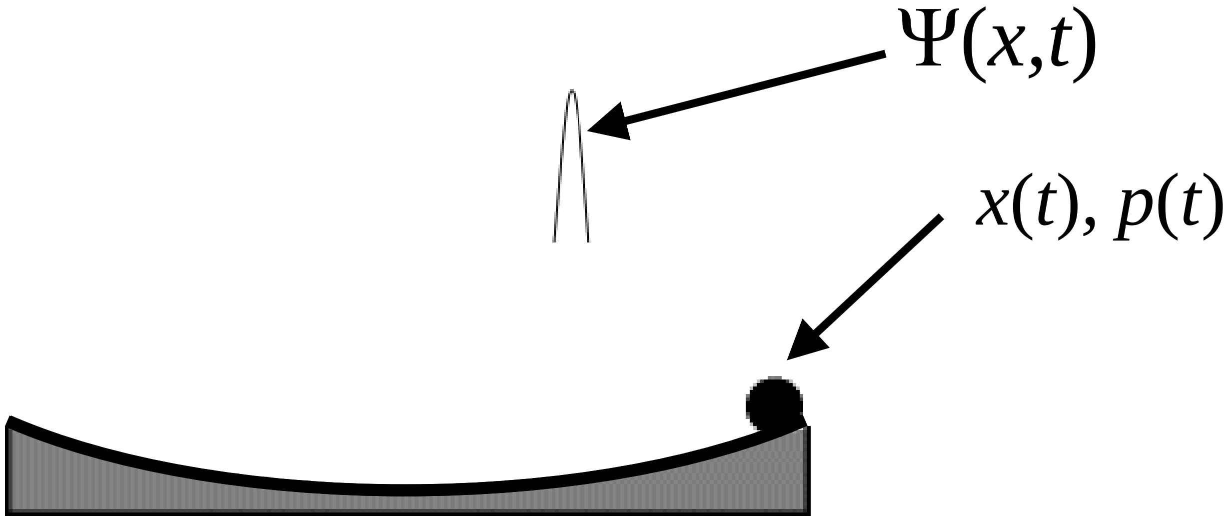
$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t)\right]\Psi(\mathbf{r},t)$$



- The wavefunction $\Psi(r)$ is used to make probabilistic predictions.
  - e.g. Probability of finding a particle at r is $|\Psi(r)|^2$
- The probability of *anything* measurable can be predicted from the $\Psi(r)$ by the **‘Born Rule’** (e.g. energy, momentum, etc.)

The Schrodinger Equation

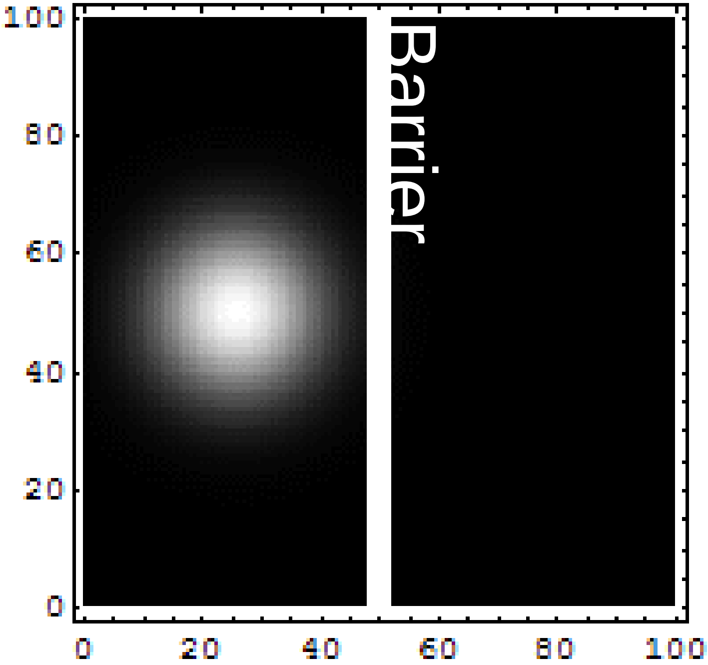
$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t)\right]\Psi(\mathbf{r},t)$$



- Along with the wavefunction, the Schrodinger Equation allows us to predict how a system changes in time.

The Schrodinger Equation

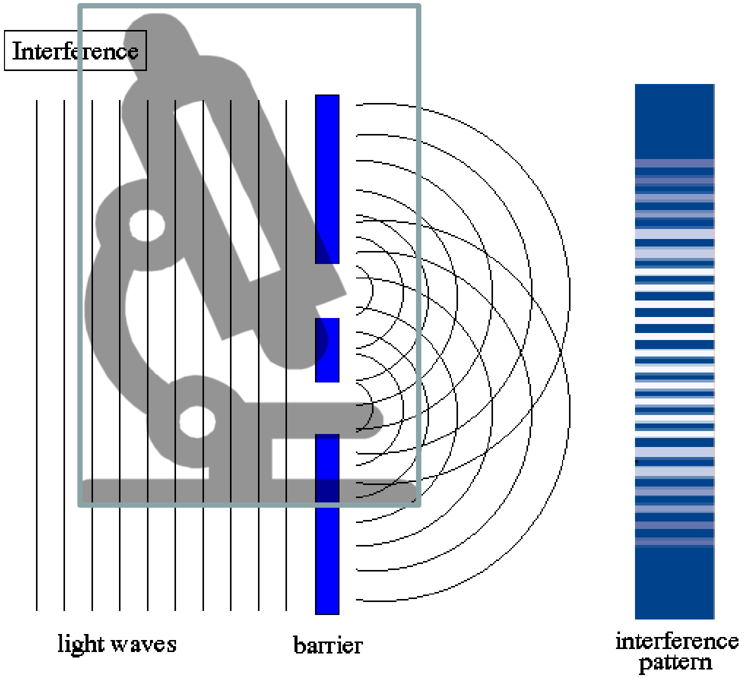
$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t)\right]\Psi(\mathbf{r},t)$$

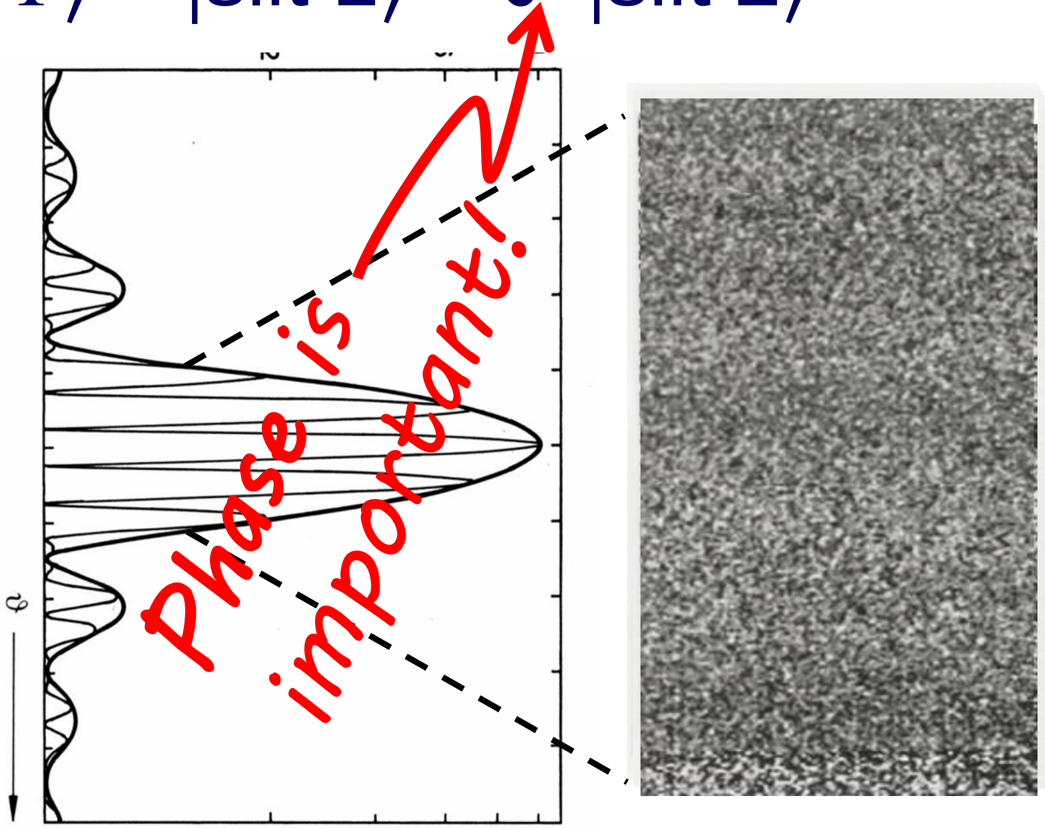


- Some predicted phenomena (e.g. tunneling) don’t happen in classical physics

Weird things about the wavefunction:
1. Wave and Particle-like Behaviour

$$\Psi(x) = |\Psi(x)|\cdot e^{i\varphi(x)} \quad \text{or} \quad |\Psi\rangle = |\text{slit } 1\rangle + e^{i\varphi}|\text{slit } 2\rangle$$



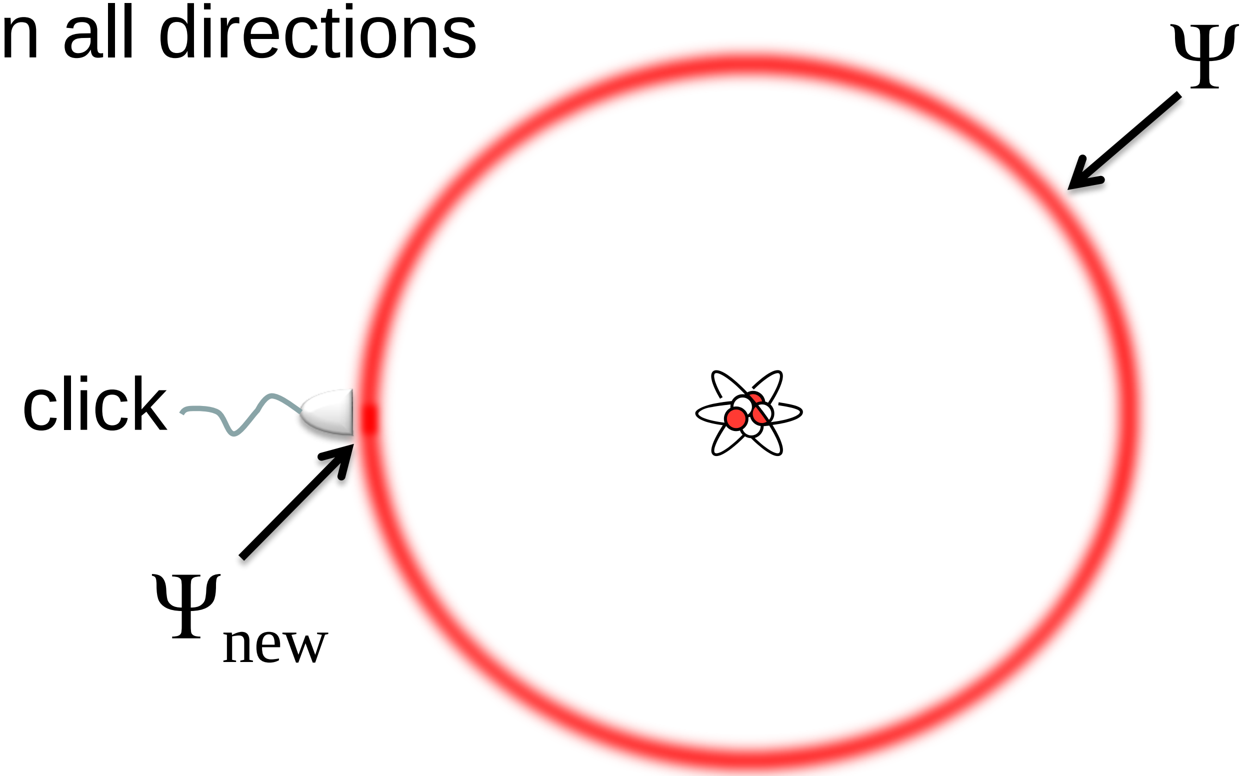


Phase is important!

- The wavefunction interferes just as though it were a real wave (e.g. a water wave)
- Even a single particle has wave-like behaviour!

Weird things about the wavefunction: 2. Measurement

Consider an atom that decays and can emit in all directions



- Ψ 'Collapses' instantaneously to Ψ_{new} → Faster than light?
- Non-deterministic, Probabilistic, not in the Schrodinger Eq.

Great minds argue about Quantum Physics

colourized by pastincolour.com

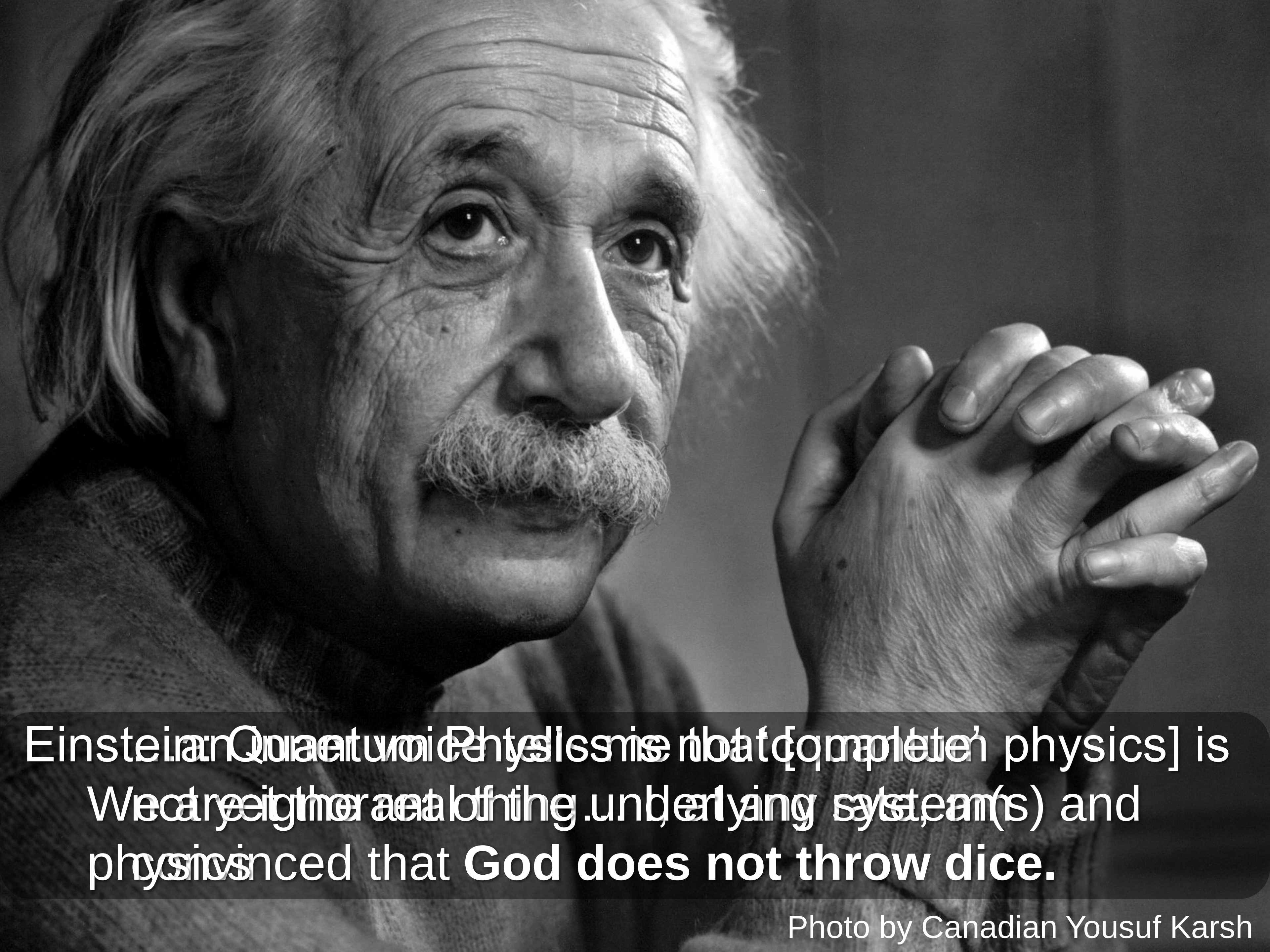
SOLVAY CONFERENCE 1927

A. PICARD E. HENRIOT P. EHRENFEST Ed. HERSEN Th. DE DONDER E. SCHRÖDINGER E. VERSCHAFFELT W. PAULI W. HEISENBERG R.H FOWLER L. BRILLOUIN

P. DEBYE M. KNUDSEN W.L. BRAGG H.A. KRAMERS P.A.M. DIRAC A.H. COMPTON L. de BROGLIE M. BORN N. BOHR

I. LANGMUIR M. PLANCK Mme CURIE H.A. LORENTZ A. EINSTEIN P. LANGEVIN Ch.E. GUYE C.T.R. WILSON O.W. RICHARDSON

Absents : Sir W.H. BRAGG, H. DESLANDRES et E. VAN AUBEL



Einstein: Quantum Physics is not complete

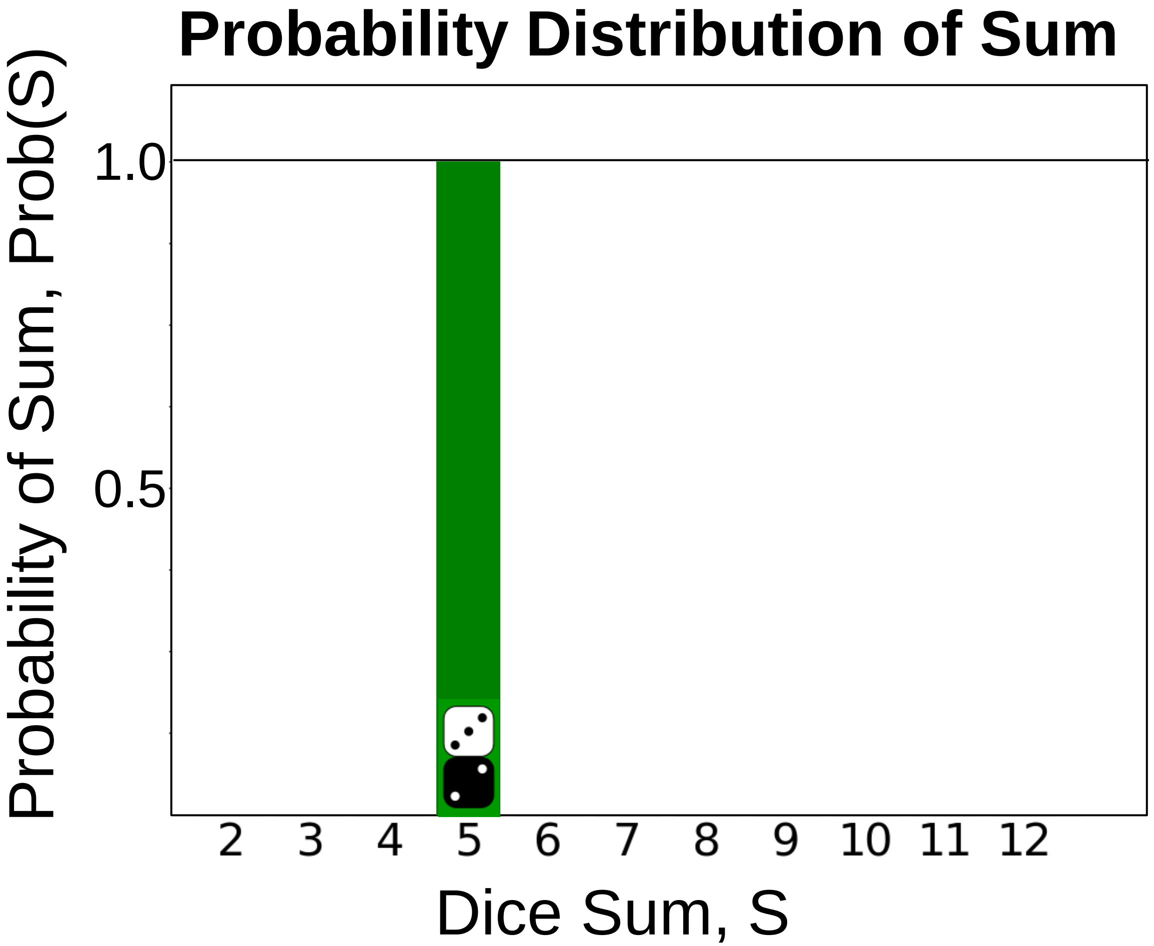
We are ignorant of the underlying system(s) and physics

…an inner voice tells me that ‘[quantum physics] is not yet the real thing…, I, at any rate, am convinced that **God does not throw dice.**

Photo by Canadian Yousuf Karsh

Is Ψ a statistical probability distribution?

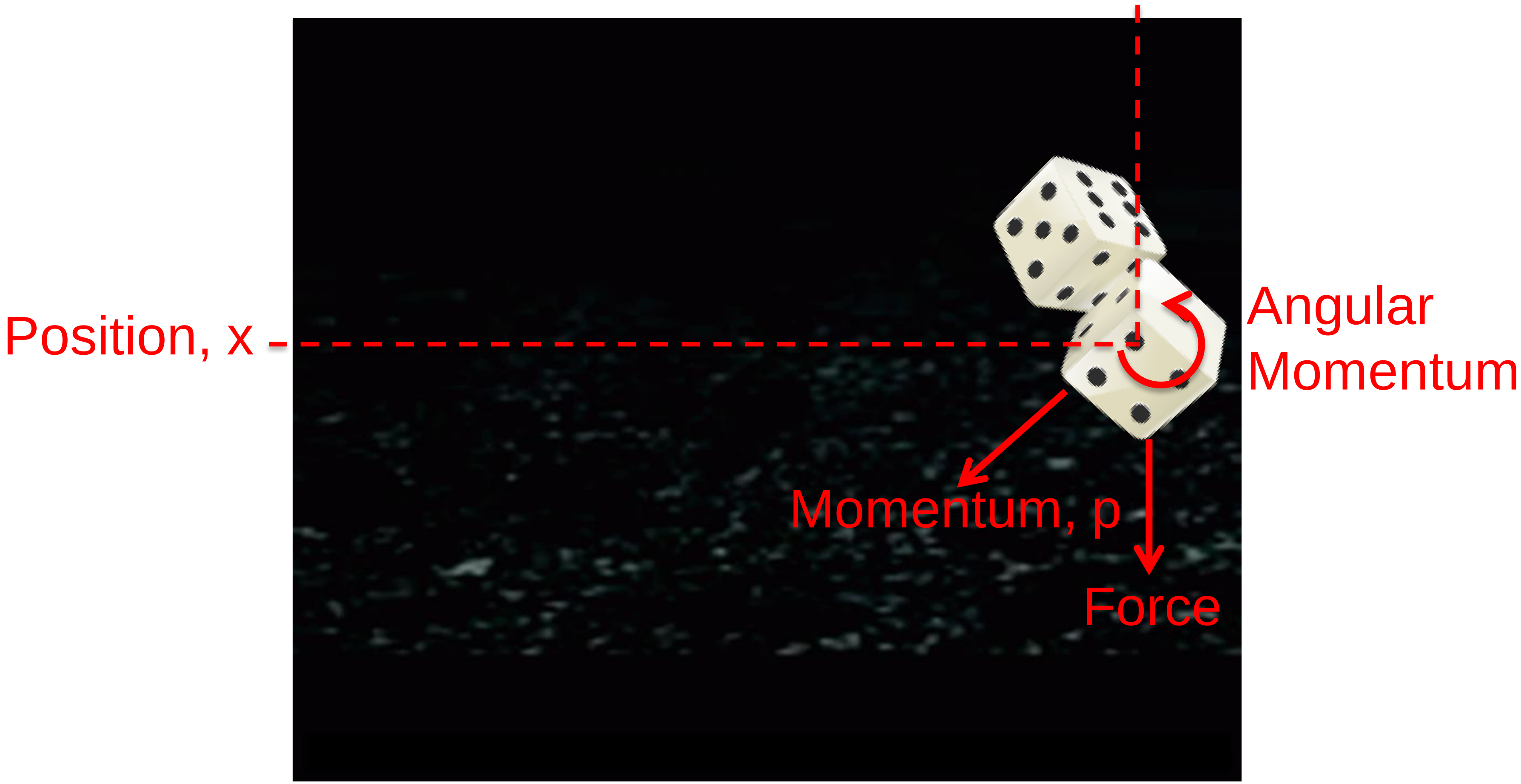
Consider rolling two dice:



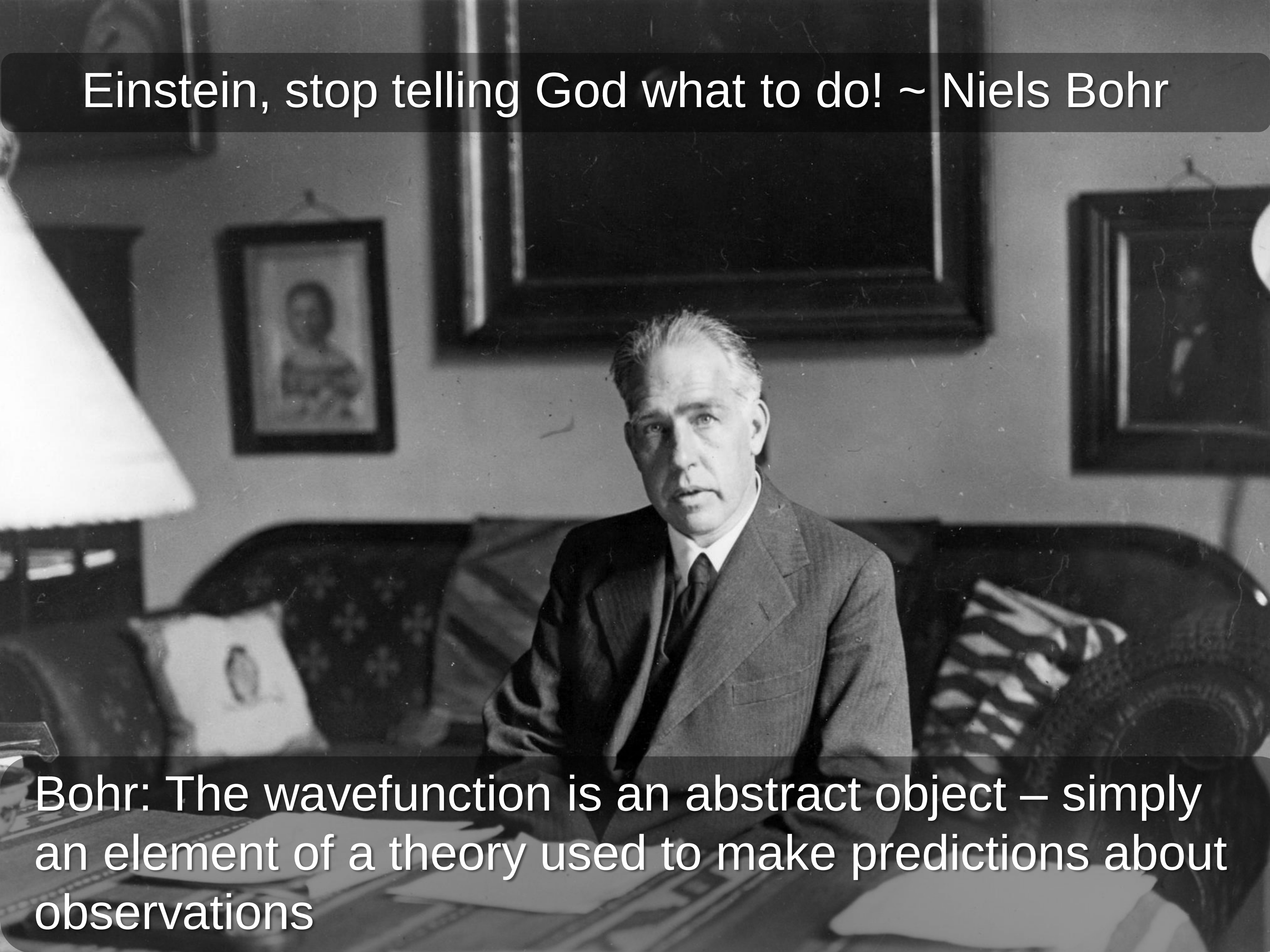
- A roll results in a particular sum S, e.g. S=5, and distribution collapses to Prob(5)=100%.
  - ➢ Collapse is no longer weird, nonlocal, or unphysical

Is Ψ a statistical probability distribution?

Consider rolling two dice:

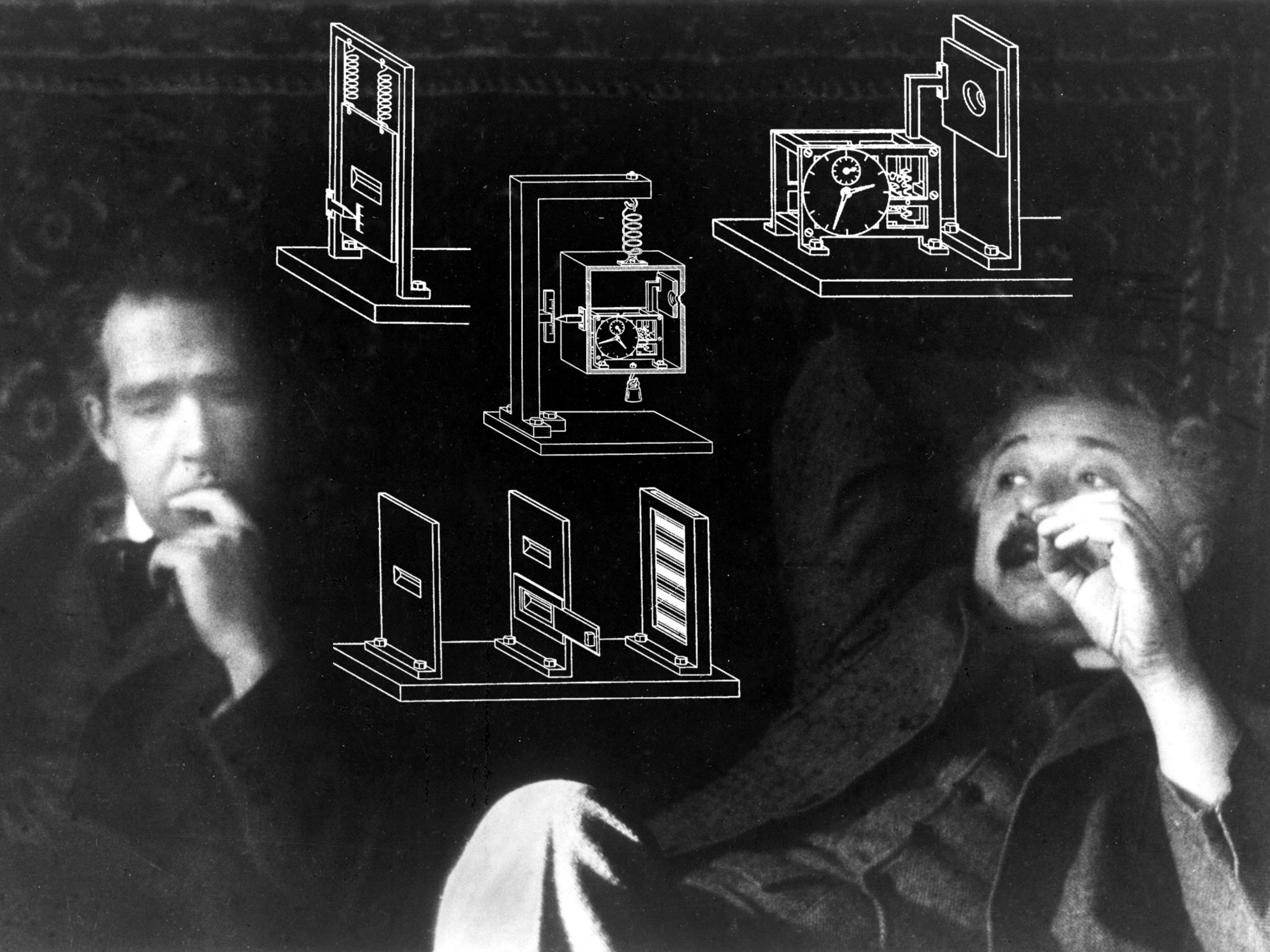


- In any given dice roll, one can predict the dice sum.
  - The underlying physics is deterministic
  - Ψ is the result of ignorance of the exact state of the dice

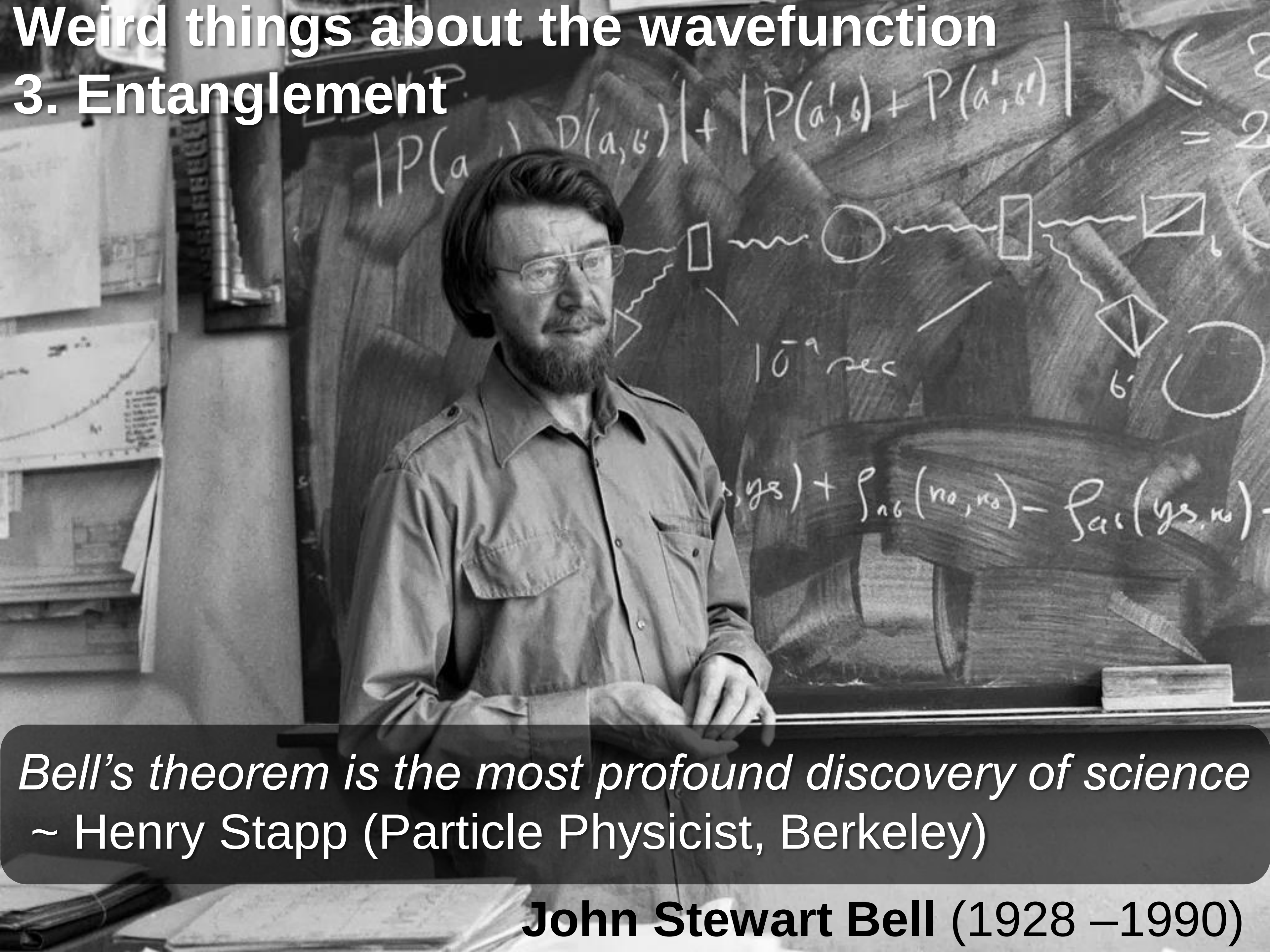


Einstein, stop telling God what to do! ~ Niels Bohr

Bohr: The wavefunction is an abstract object – simply an element of a theory used to make predictions about observations



Weird things about the wavefunction
3. Entanglement



Bell's theorem is the most profound discovery of science
~ Henry Stapp (Particle Physicist, Berkeley)

John Stewart Bell (1928 –1990)

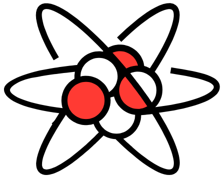
Bell’s Theorem

- Violation of the Bell inequality shows that nature is either (or both)
  - Nonlocal: things can instantaneously affect other far away things.
  - Not real: No fixed pre-existing properties that determine the results of measurements.

Bell’s Theorem

- Violation of the Bell inequality shows that nature is either (or both)
  - Nonlocal: things can instantaneously affect other far away things.
  - Not real: No fixed pre-existing properties that determine the results of measurements.

Quantum entangled systems can violate the inequality:



The spins are perfectly opposite in all directions:

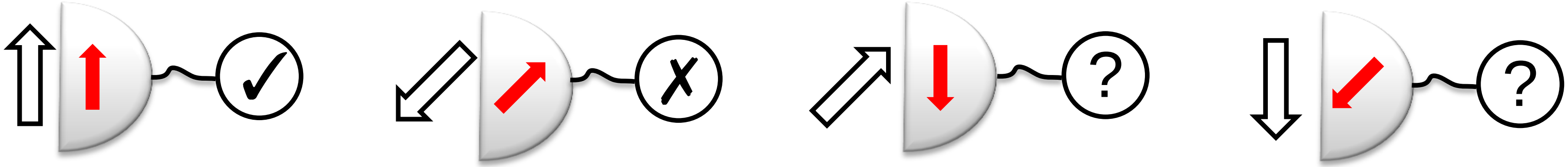
$$|\Psi\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle = |\rightarrow\leftarrow\rangle - |\leftarrow\rightarrow\rangle = |\nwarrow\searrow\rangle - |\searrow\nwarrow\rangle \neq |?\rangle_1 |?\rangle_2$$

Entangled: No wavefunction for one particle by itself

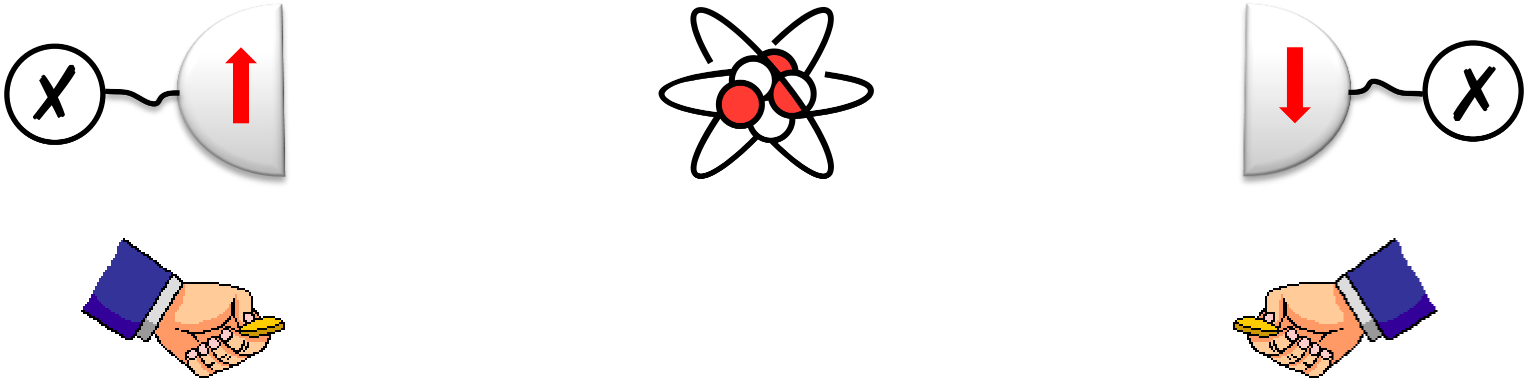
Bell’s Theorem

Detectors are set to detect particles of certain direction

Consider four cases:



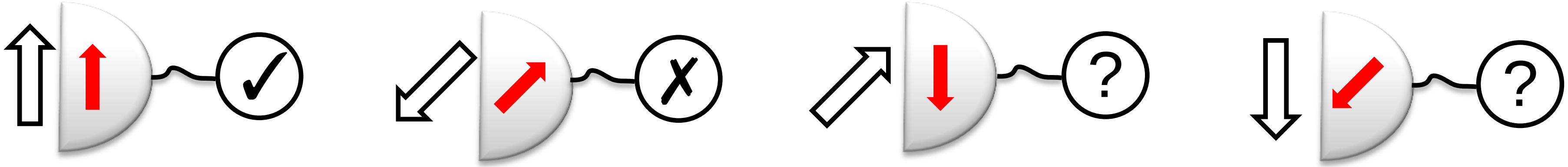
1. Randomly set the detector directions
2. Do this outside the other detector’s light cone



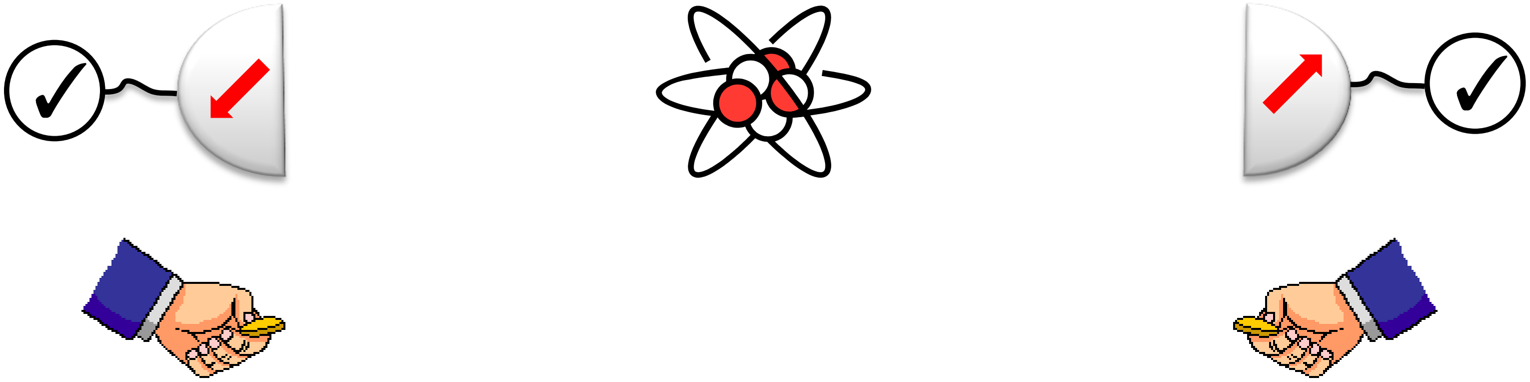
Bell’s Theorem

Detectors are set to detect particles of certain direction

Consider four cases:



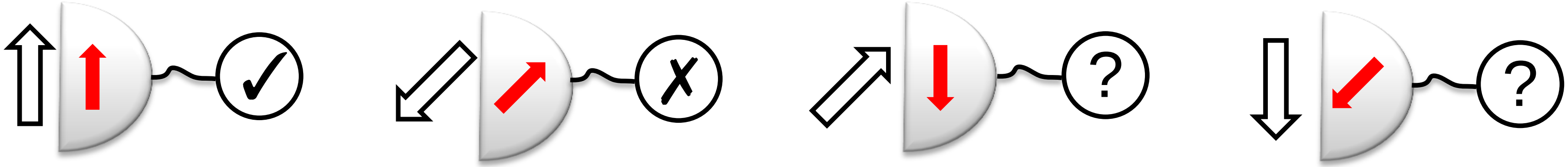
1. Randomly set the detector directions
2. Do this outside the other detector’s light cone



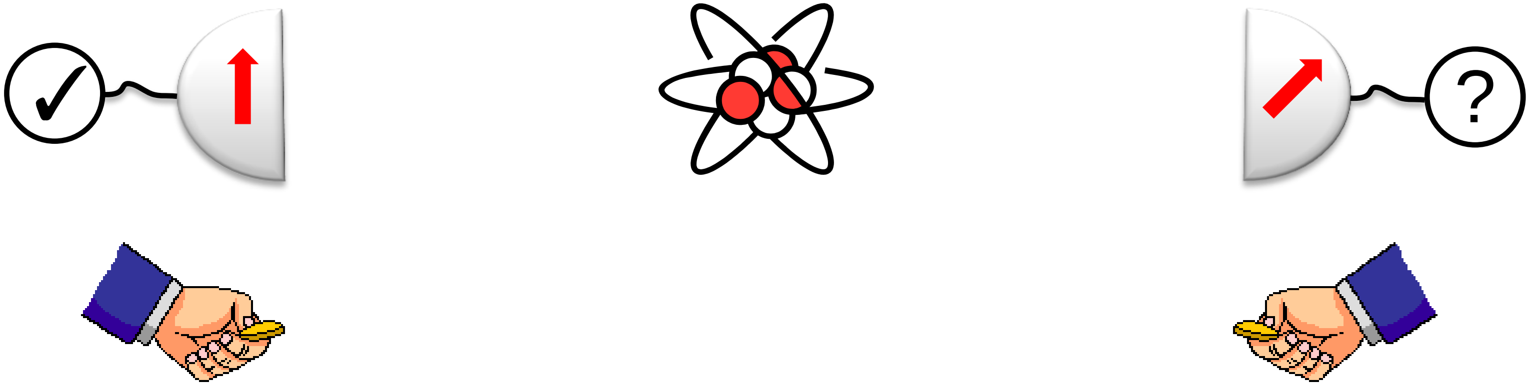
Bell’s Theorem

Detectors are set to detect particles of certain direction

Consider four cases:



1. Randomly set the detector directions
2. Do this outside the other detector’s light cone



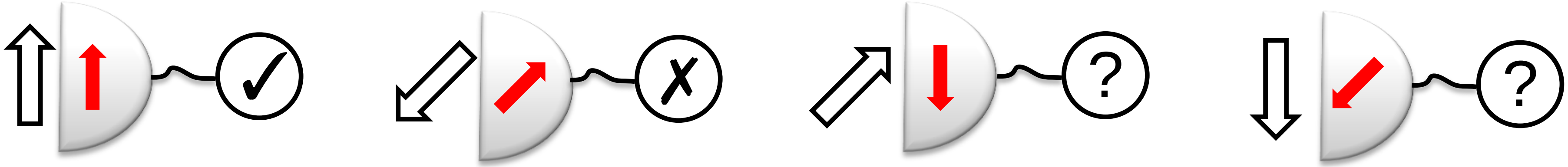
Perfect correlations: e.g. even/odd digits of 3.14159265

Perfect randomness: e.g. flip a second coin for ✗ or ✓

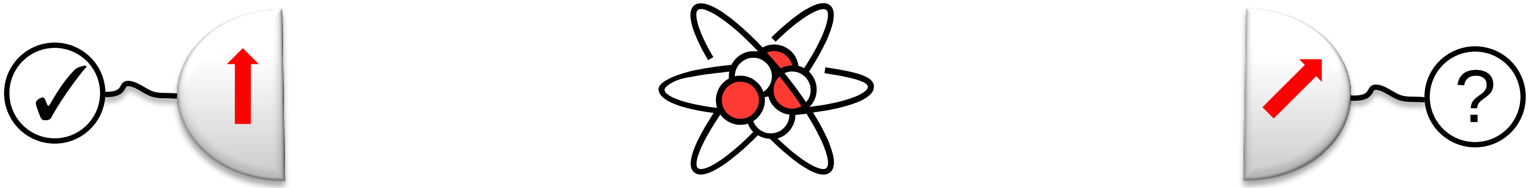
Bell’s Theorem

Detectors are set to detect particles of certain direction

Consider four cases:



1. Randomly set the detector directions
2. Do this outside the other detector’s light cone



$$|C(a,b)-C(a,c)|\leq 1+C(b,c)$$

Local realistic theories can produce either perfect correlations or perfect randomness… but not both.

∴ The wavefunction is not locally real

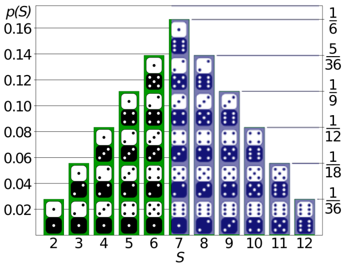
Recent Progress in understanding Ψ

Quantum State Cannot be Interpreted Statistically

Pusey, Barrett & Rudolph, Nature Physics, 2011.

Marginally different wavefunctions correspond to completely distinct underlying statistical states.

$$\text{if } \langle \Psi | \Psi \rangle \neq 1 \text{ then}$$

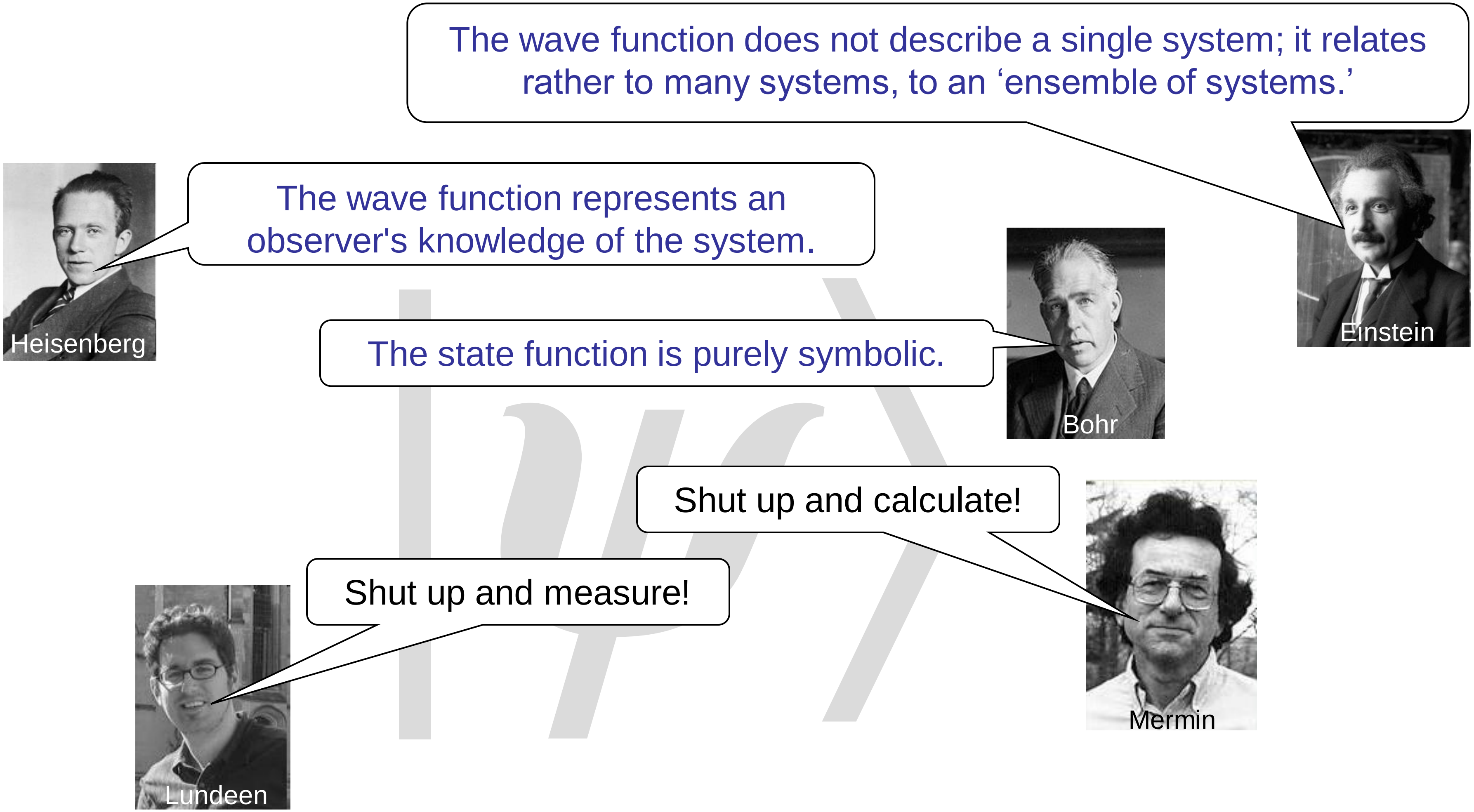


No extension of quantum theory can have improved predictive power,

Colbeck & Renner, Nature Comm. 2011

Assuming the Born Rule is correct, nature is sufficiently constrained by it to not leave room for new or better experimental predictions.

What is the wavefunction?



The wave function does not describe a single system; it relates rather to many systems, to an ‘ensemble of systems.’

The wave function represents an observer's knowledge of the system.

The state function is purely symbolic.

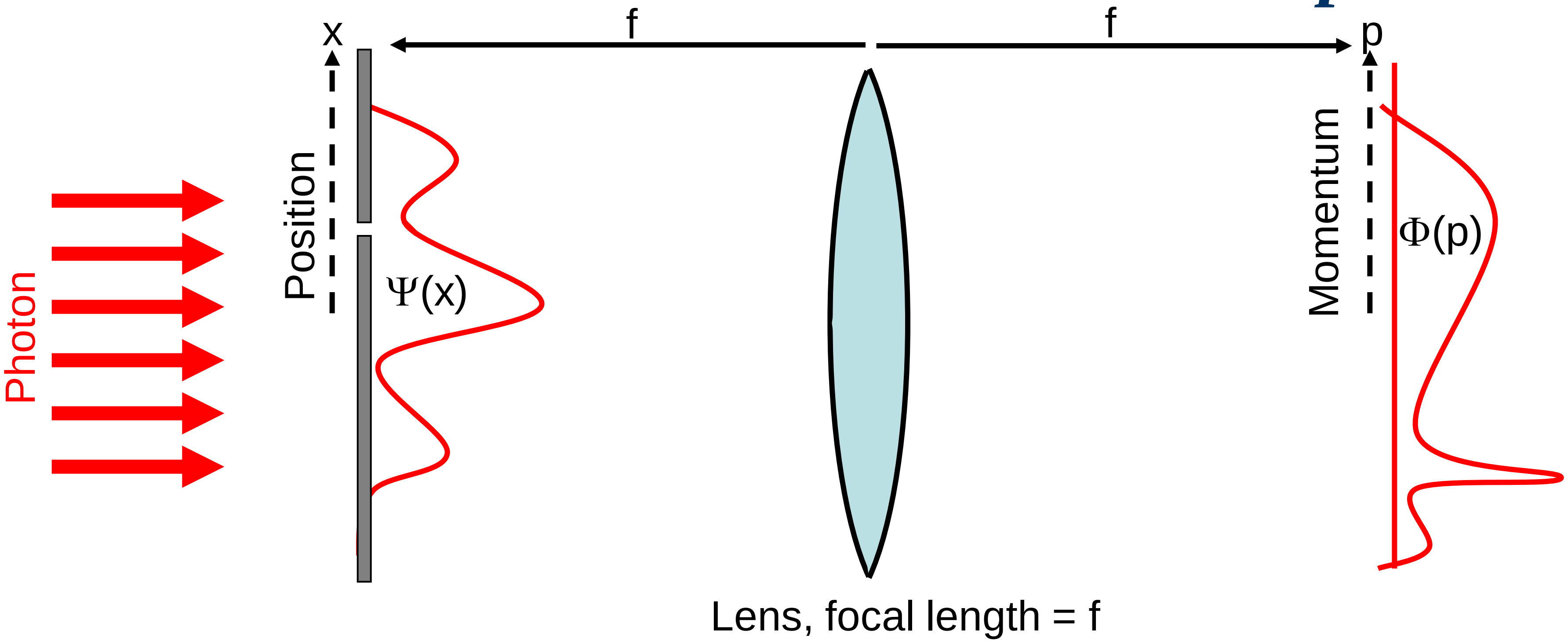
Shut up and calculate!

Shut up and measure!

No-Cloning Theorem: one cannot copy a particle’s wavefunction

Corollary: It is impossible to determine an arbitrary wavefunction of a single particle.

Simultaneous Measurement of x and p

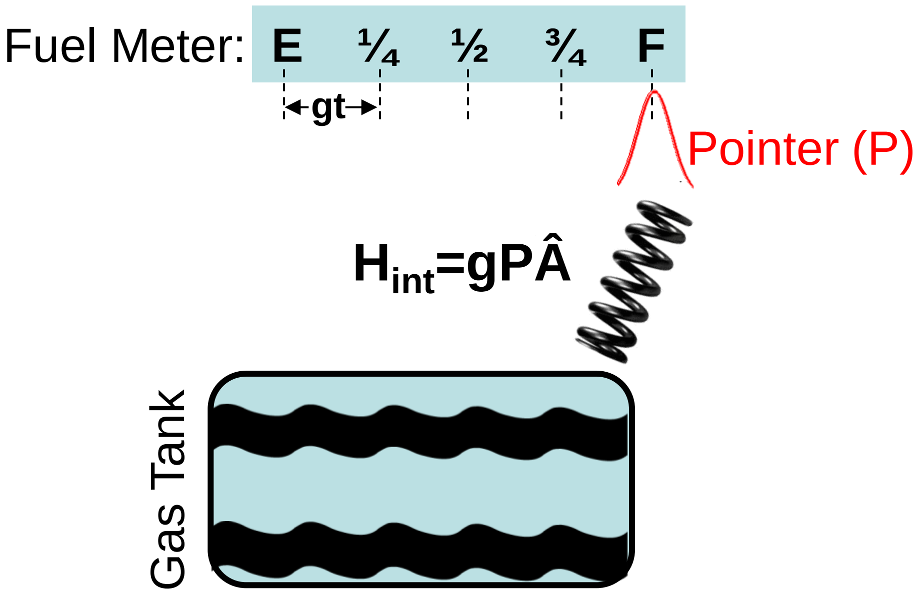


- Can easily measure Prob(x)=$|\Psi(x)|^2$ and then Prob(p)= $|\Phi(p)|^2$
- We don't see the phase, i.e. the θ in $\Psi=|\Psi|e^{i\theta}$
- Measure x and we cause $\Delta p \to \infty$
  - ➢ "Heisenberg Uncertainty **Relation**"
  - ➢ Can not know x and p perfectly at the same time

Why not gently measure x and then strongly measure p?

Quantum Measurement

Strong Measurement



$H_{int}=gP\hat{A}$

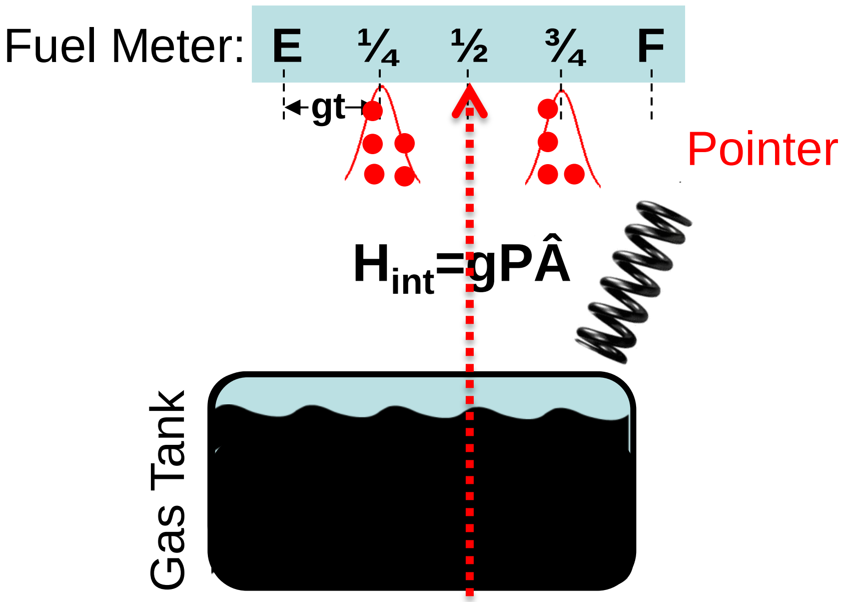
System+Pointer=$\sum c_i|a_i\rangle|P_i\rangle$

Model both the measured system and the measurement apparatus as quantum systems.

e.g. The pointer needle on a fuel gauge has a wavefunction and so does the gas tank.

Quantum Measurement

Strong Measurement



System+Pointer=$\sum c_i|a_i\rangle|P_i\rangle$

System = |¾

Average Value of **A**:

$\langle\psi|\hat{A}|\psi\rangle$

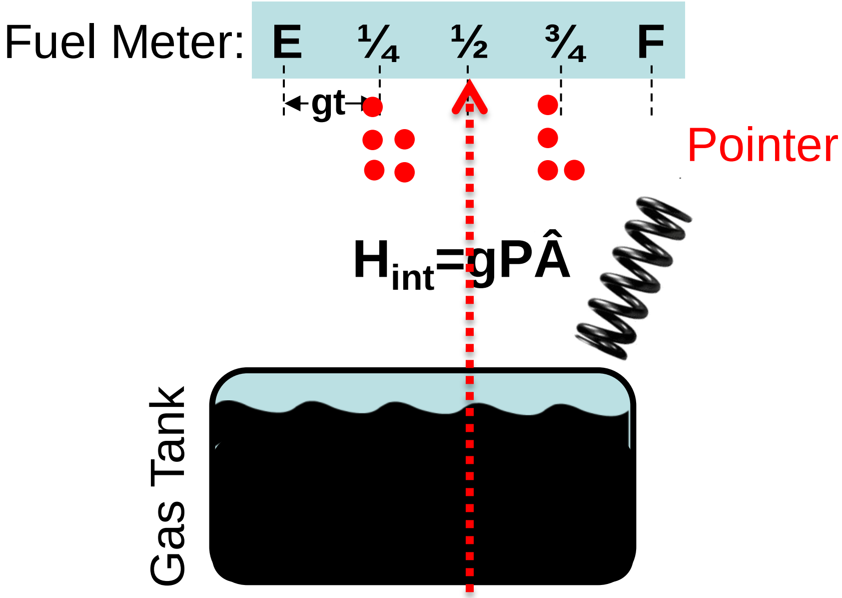
Born Rule!

Model both the measured system and the measurement apparatus as quantum systems.

e.g. The pointer needle on a fuel gauge has a wavefunction and so does the gas tank.

Quantum Measurement

Strong Measurement



$H_{int} = gP\hat{A}$

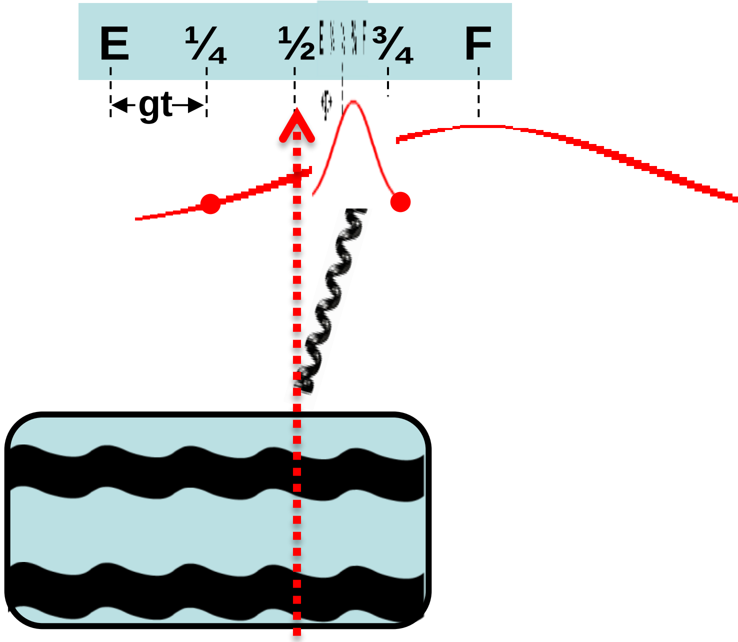
System=$|\tfrac{3}{4}\rangle$

Average Value of **A**:

$\langle\psi|\hat{A}|\psi\rangle$

g<<1

Weak Measurement



System, $|\psi\rangle = \sum c_i|a_i\rangle$

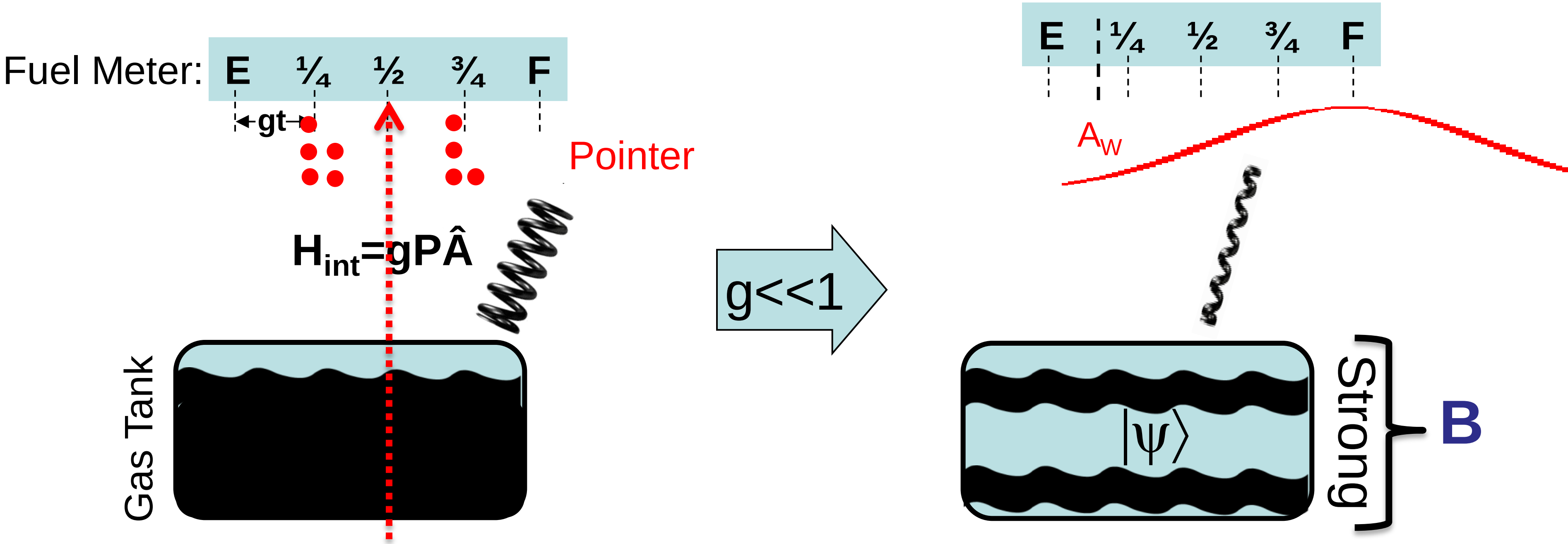
Average Value of **A**:

$\langle\psi|\hat{A}|\psi\rangle$

Quantum Measurement

Strong Measurement

Weak Measurement



System=$|3/4\rangle$

Average Value of **A**:

$\langle\psi|\hat{A}|\psi\rangle$

In the cases where result of **B** is b

Average Value of **A**:

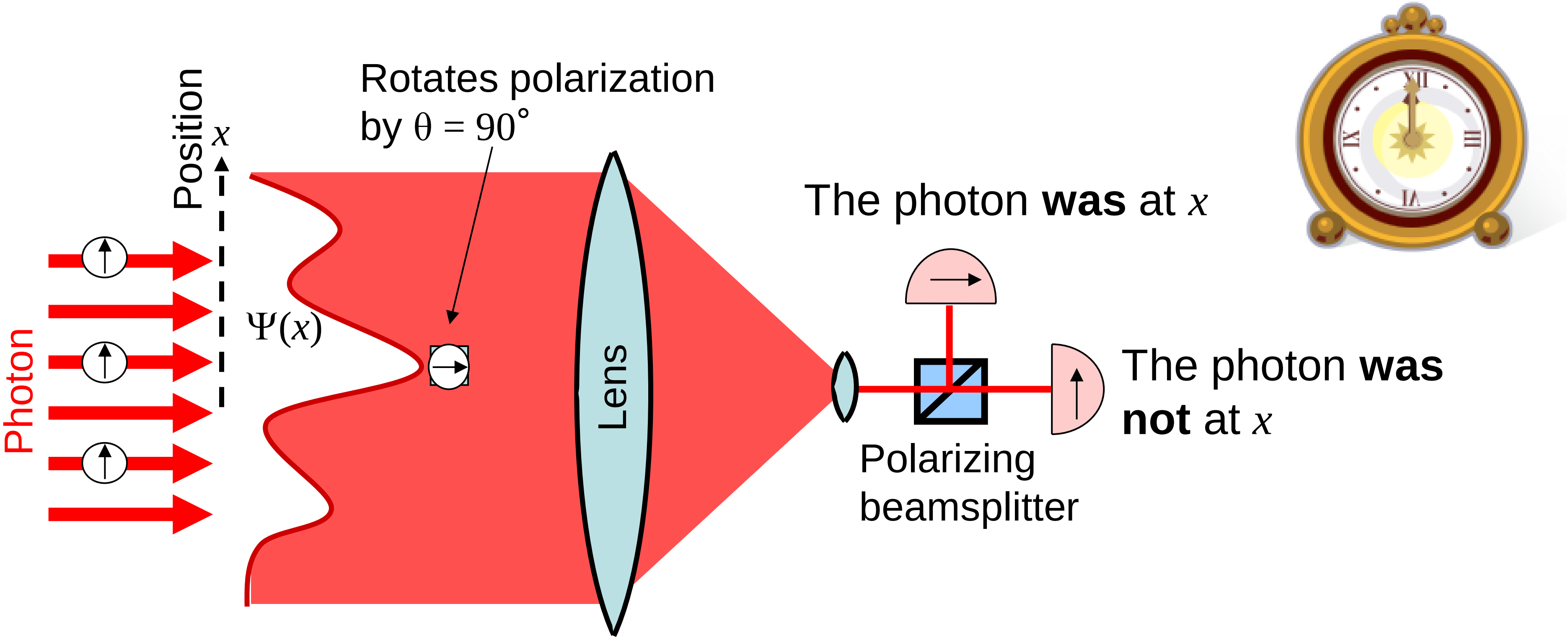
$$A_W=\frac{\langle b|A|\psi\rangle}{\langle b|\psi\rangle}$$

Real part of A_W is the position shift of the pointer

Imaginary part of A_w is the momentum shift of the pointer

Strong Measurement Example

- **Consider a strong measurement of position, e.g.** $|x\rangle\langle x| \equiv \pi$



Rotates polarization by $\theta = 90^\circ$

The photon **was** at x

The photon **was not** at x

Polarizing beamsplitter

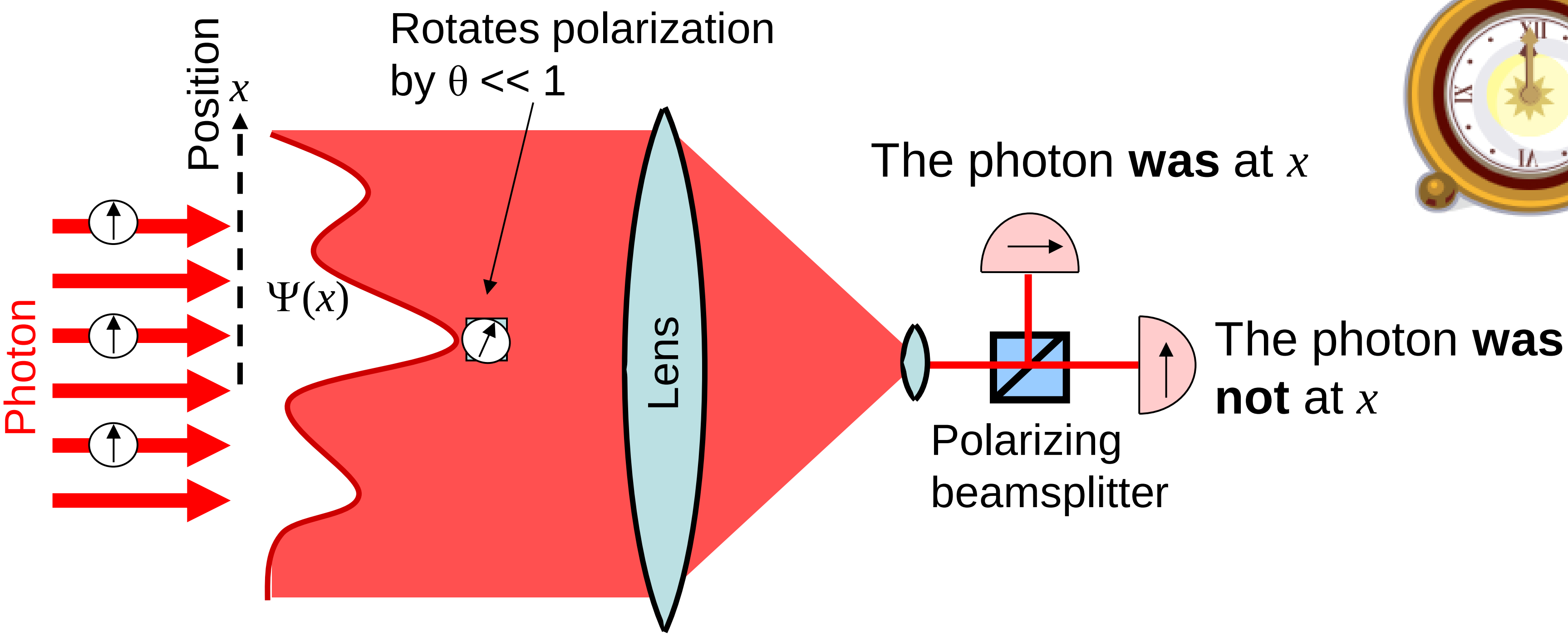
- **The average result of a strong measurement:**

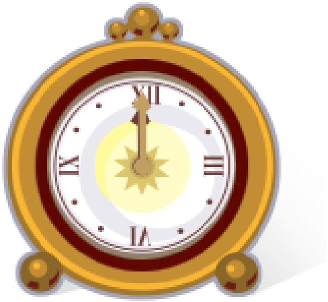
$$\langle \boldsymbol{\pi} \rangle = \langle\psi||x\rangle\langle x||\psi\rangle = |\psi(x)|^2 = \mathrm{Prob}(x)$$

$=$ the probability of finding the photon at position x

Weak Measurement Example

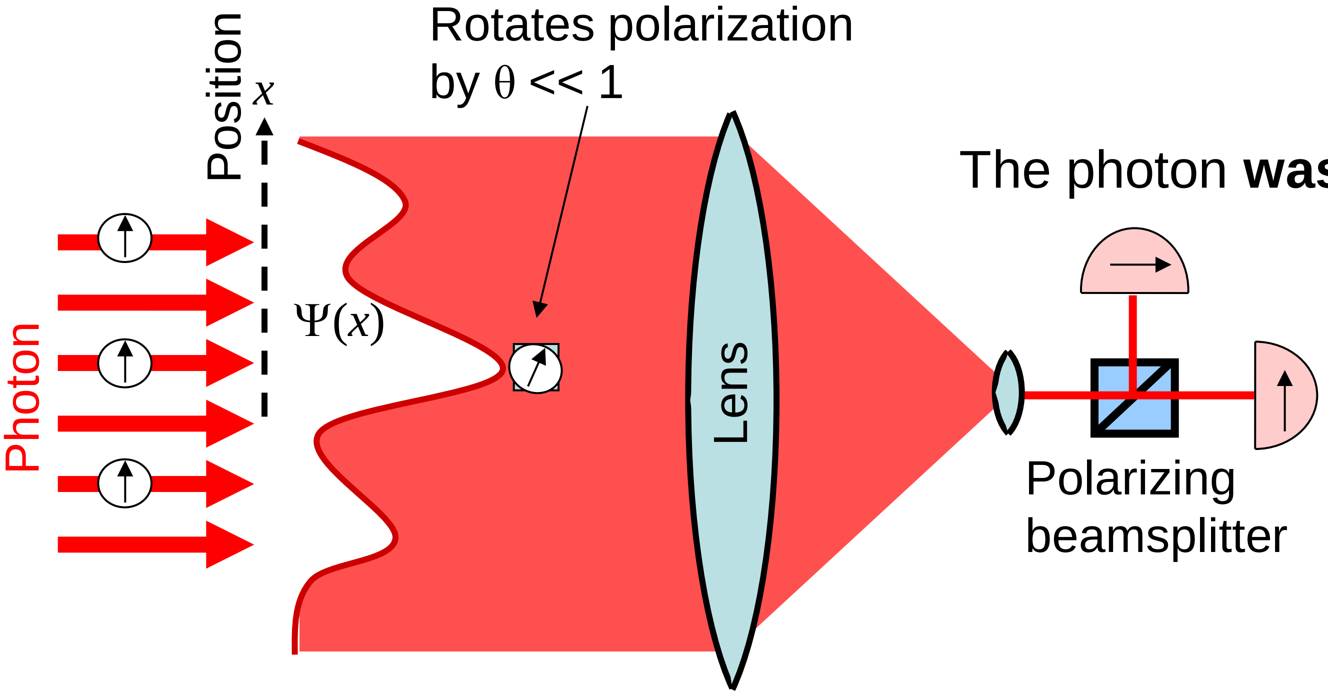
- **For a weak measurement we reduce the rotation of the polarization**





Weak Measurement Example

- **For a weak measurement we reduce the rotation of the polarization**

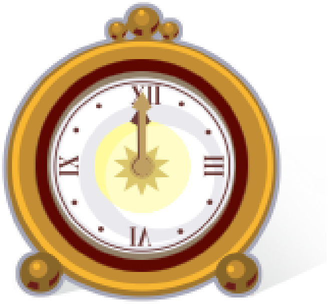


Rotates polarization by $\theta << 1$

The photon **was** at x

The photon **was not** at x

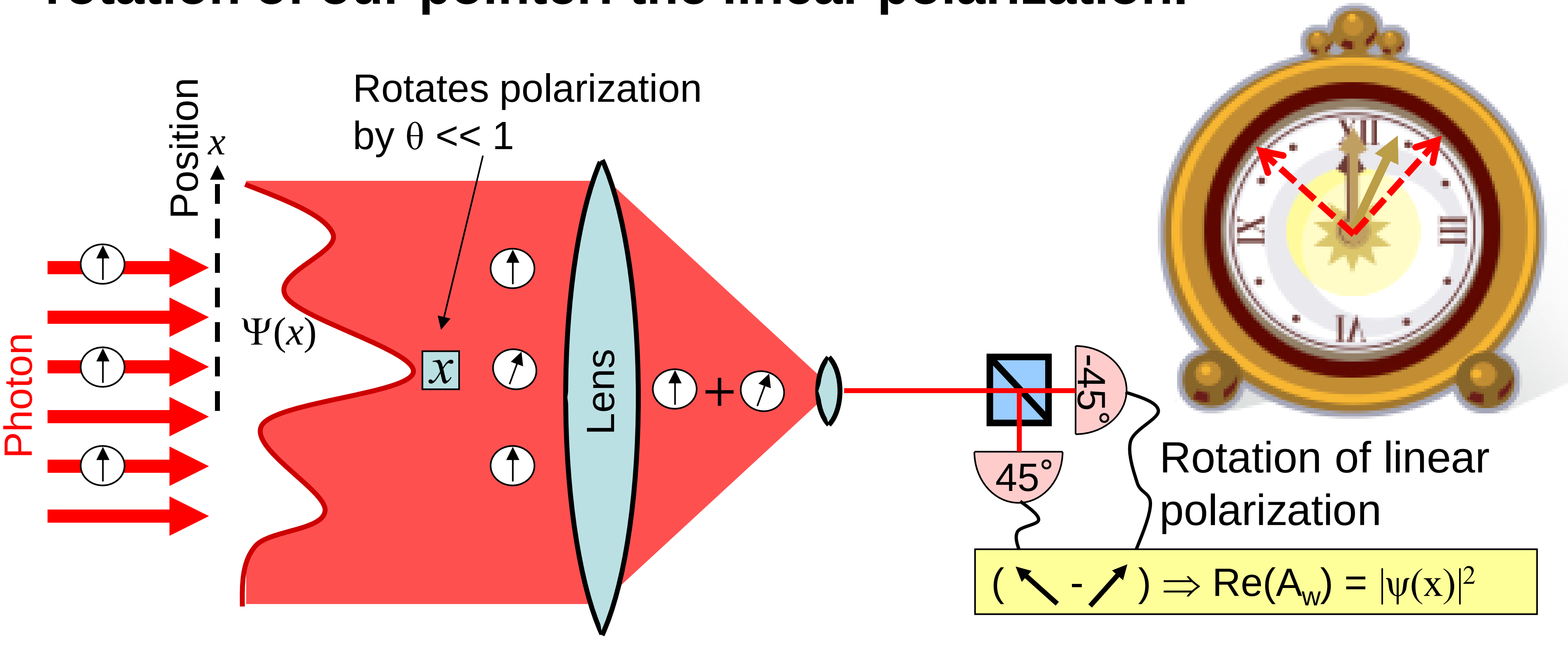
Polarizing beamsplitter



- **From a single run, we get little information – the result is random**

Weak Measurement Example

- **The average result of the weak measurement is the final rotation of our pointer: the linear polarization.**

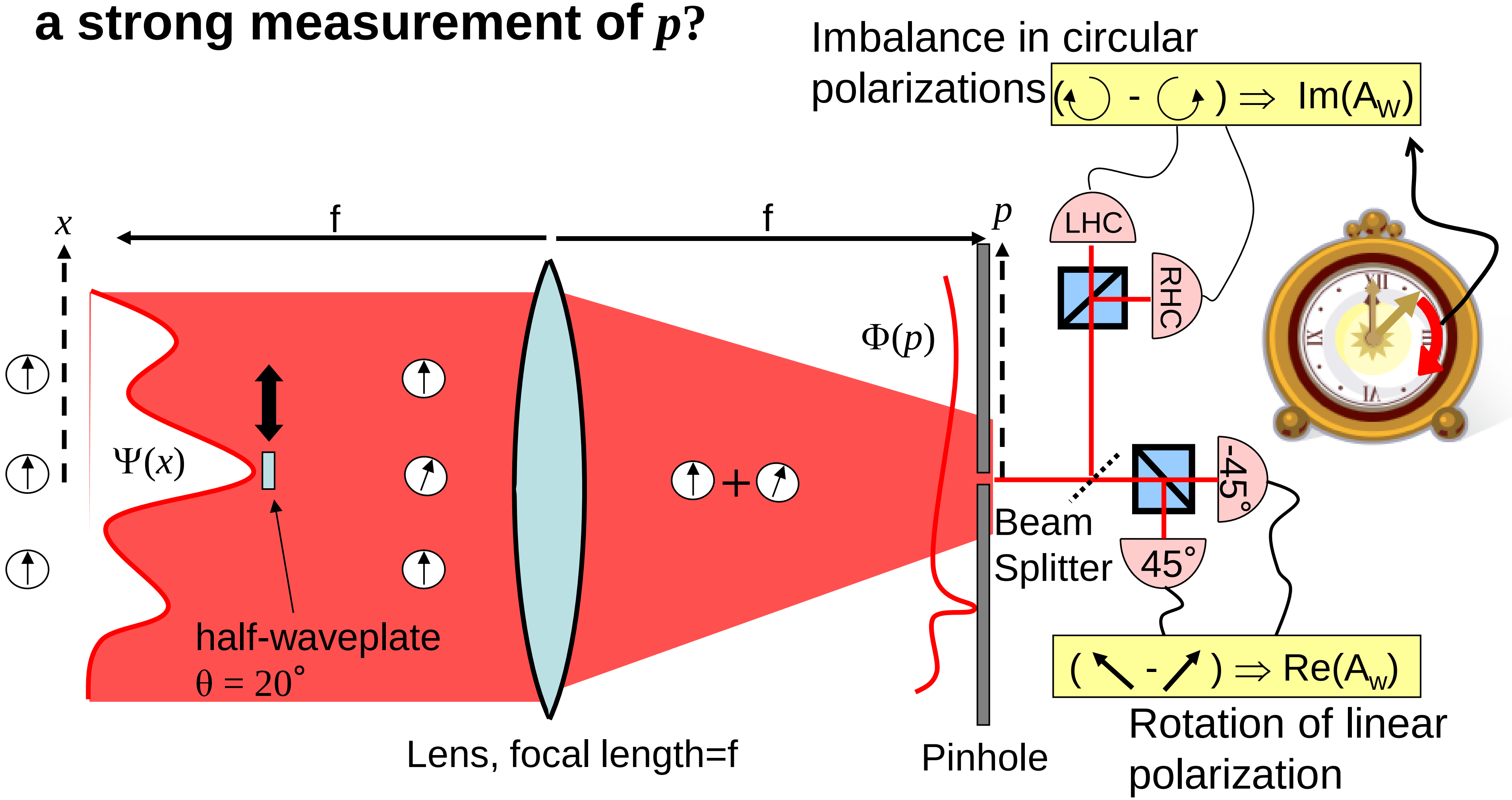


- **The average result of the weak measurement is the same as a standard (‘strong’) one:**

$$\langle \mathbf{A_w} \rangle = \mathrm{Re}(\mathbf{A_w}) = \langle\psi||x\rangle\langle x||\psi\rangle = |\psi(x)|^2 = \mathrm{Prob}(x)$$

Weak then Strong Measurement

- **What if we do a weak measurement of x, and then make a strong measurement of p?**



Imbalance in circular polarizations

(↺ - ↻) ⇒ Im(A_W)

(↖ - ↗) ⇒ Re(A_w)

Rotation of linear polarization

half-waveplate $\theta = 20°$

Lens, focal length=f

Pinhole

Beam Splitter

- **Real and Imaginary parts of the weak measurement average appear in the linear and circular polarization rotations.**

The idea

- What if we do a weak measurement of $\boldsymbol{X}$, and then make a strong measurement of $\boldsymbol{P}$?

i.e. **A** $= |x\rangle\langle x| = \pi$, Initial state= $|\psi\rangle$, Strong measurement result $P=p$

Average shift of the pointer:

$$A_W = \frac{\langle b|A|\psi\rangle}{\langle b|\psi\rangle}$$

$$\pi_w = \frac{\langle p|x\rangle\langle x|\psi\rangle}{\langle p|\psi\rangle}$$

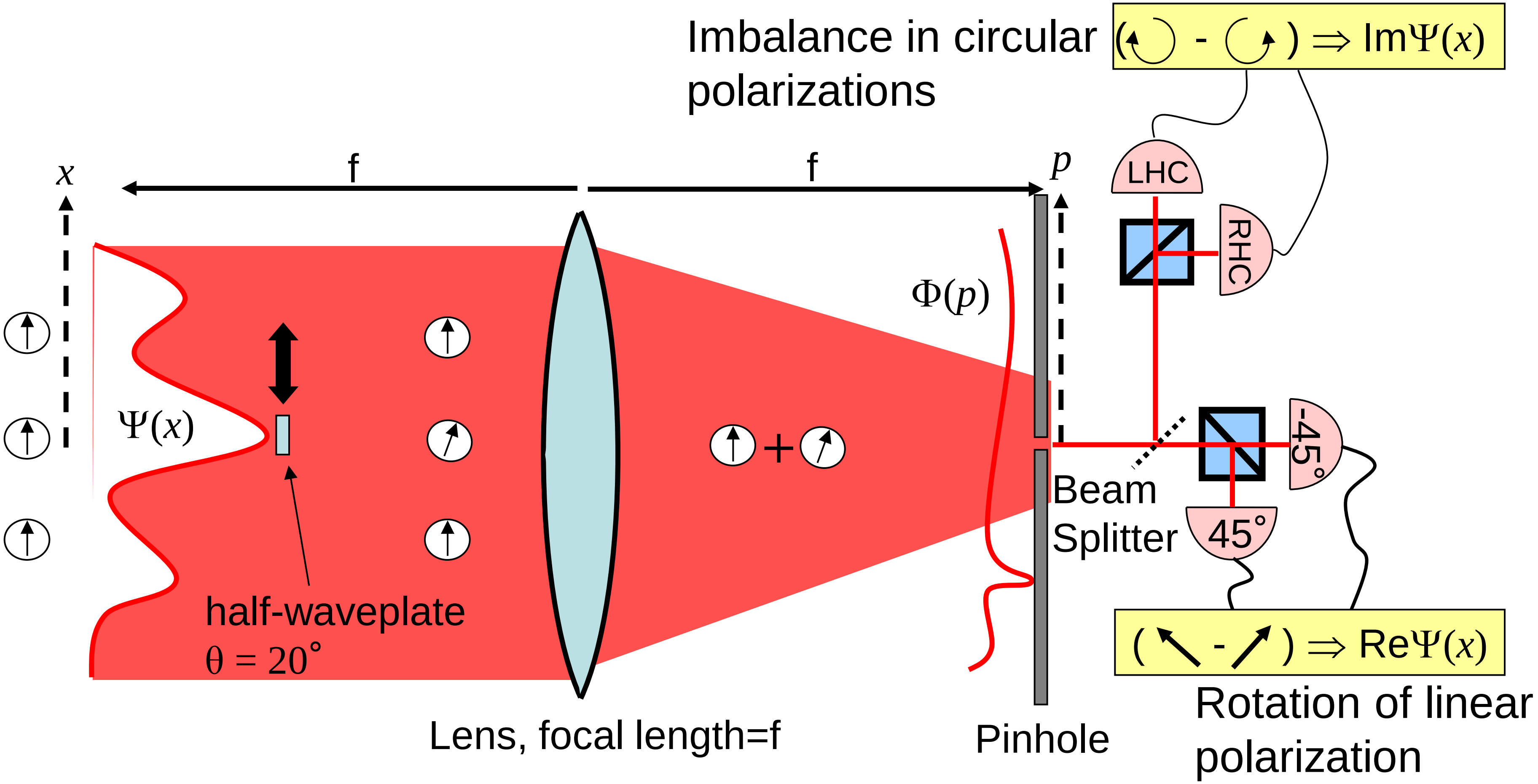
And if $p=0$,

$$\pi_w = \frac{1/\sqrt{2\pi}\cdot\langle x|\psi\rangle}{\sqrt{\mathrm{Prob}(p=0)}} = \boxed{k\cdot\psi(x)}$$

- **The average shift of the pointer (i.e. rotation of the polarization) is proportional to the wavefunction**

Direct Measurement of the Wavefunction

- Weakly measure $|x\rangle\langle x|$ then strongly measure p, and keep only the photons found with $p=0$.



Imbalance in circular polarizations

(↺ - ↻) $\Rightarrow \mathrm{Im}\Psi(x)$

half-waveplate $\theta = 20^\circ$

Lens, focal length=f

Pinhole

Beam Splitter

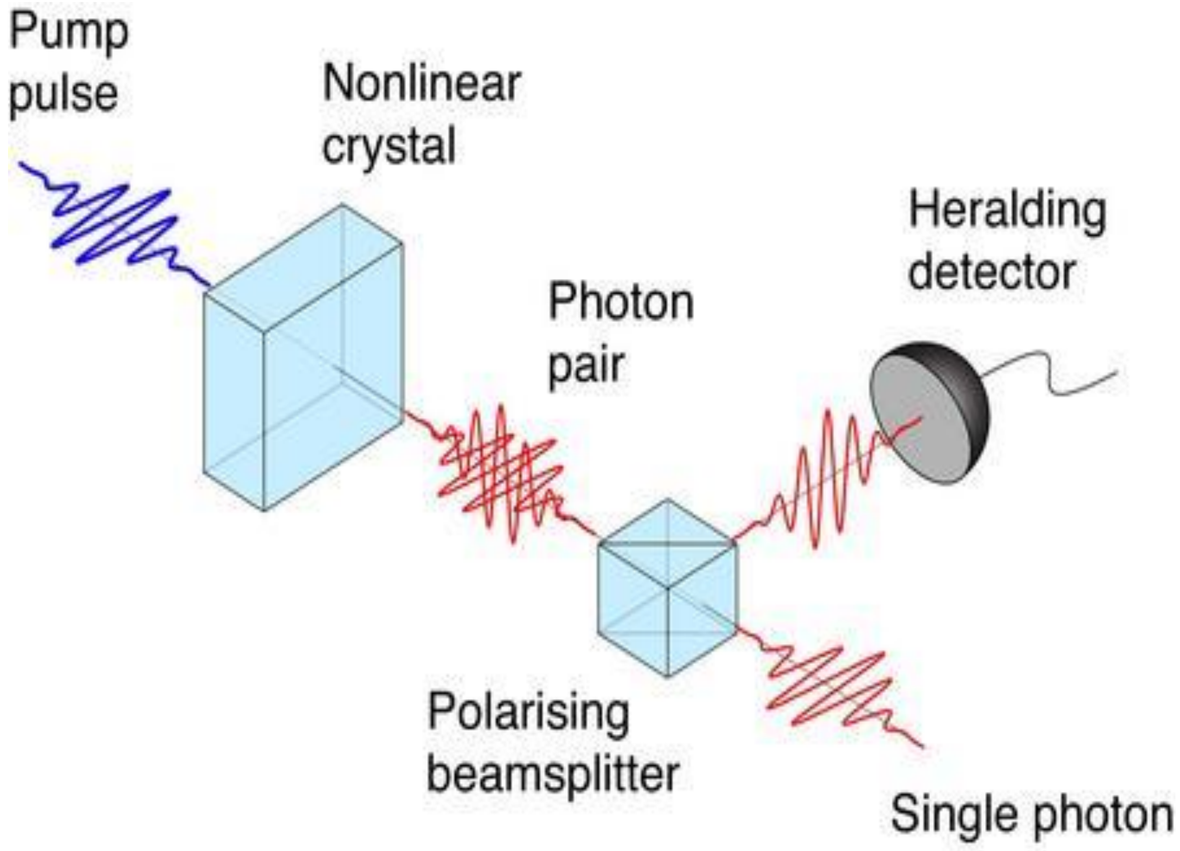
(↖ - ↗) $\Rightarrow \mathrm{Re}\Psi(x)$

Rotation of linear polarization

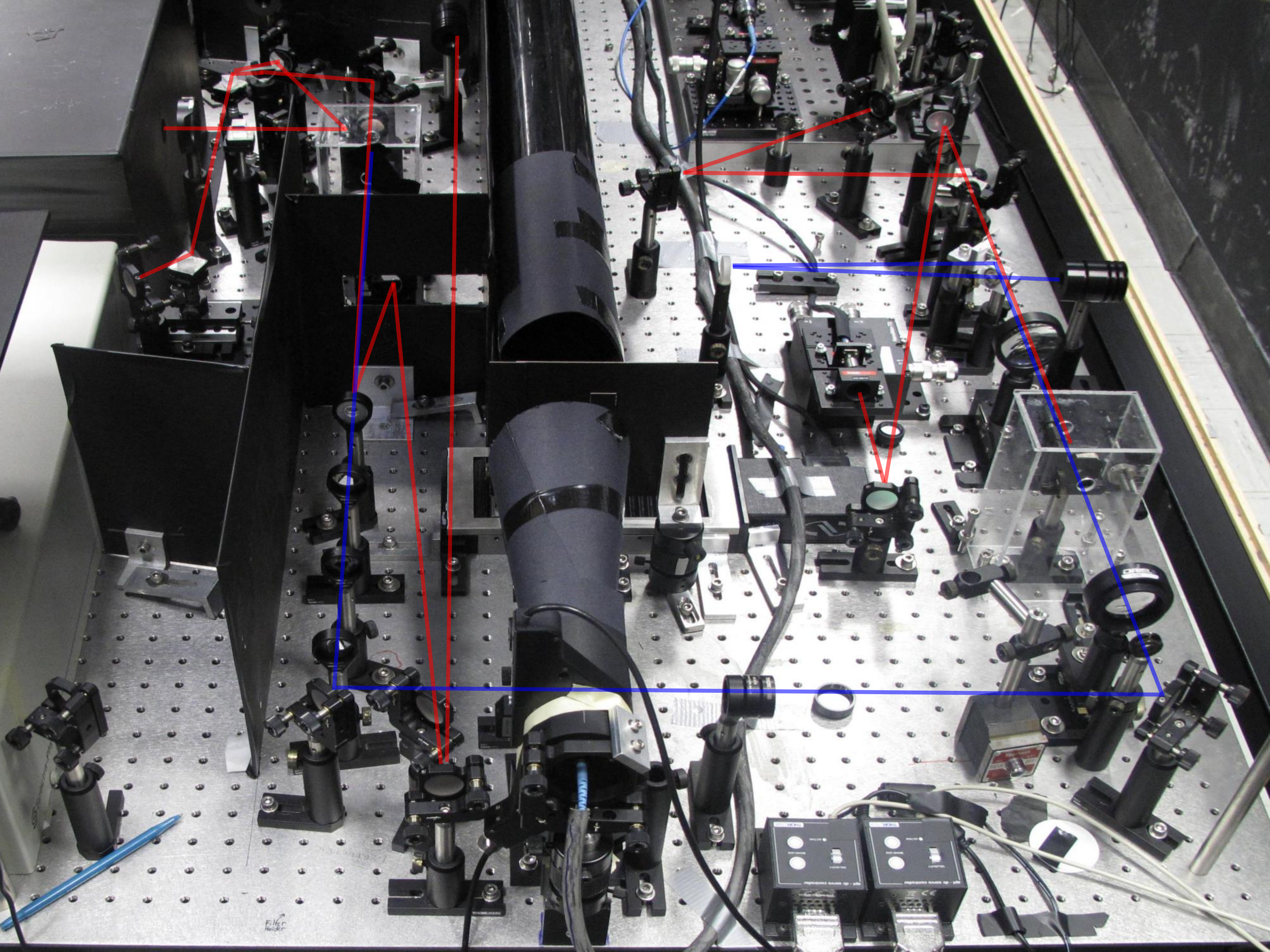
- The average result of the weak measurement is the real and imaginary components of the wavefunction

Our Source of Single Photons

- A pump photon is spontaneously converted into two lower frequency photons in a nonlinear optical material

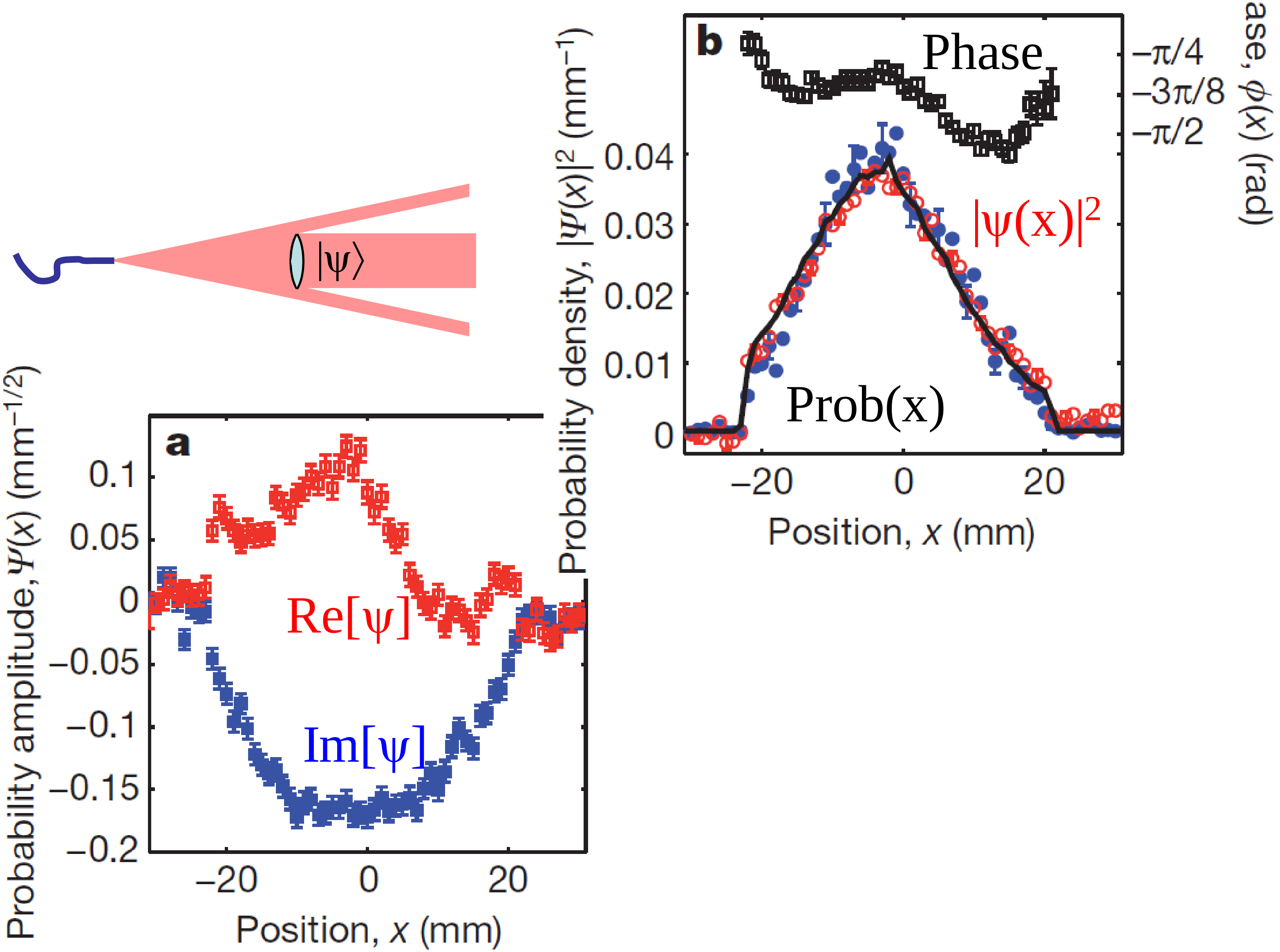


- Photons are produced rarely but always in pairs
  → Detection of one photon ‘Heralds’ the presence of its twin



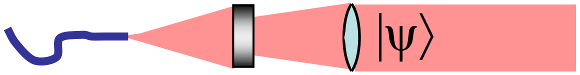
Filter holder

Direct Measurement of the Wavefunction

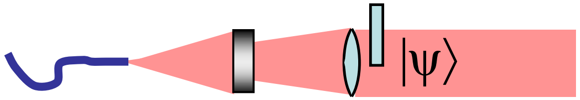


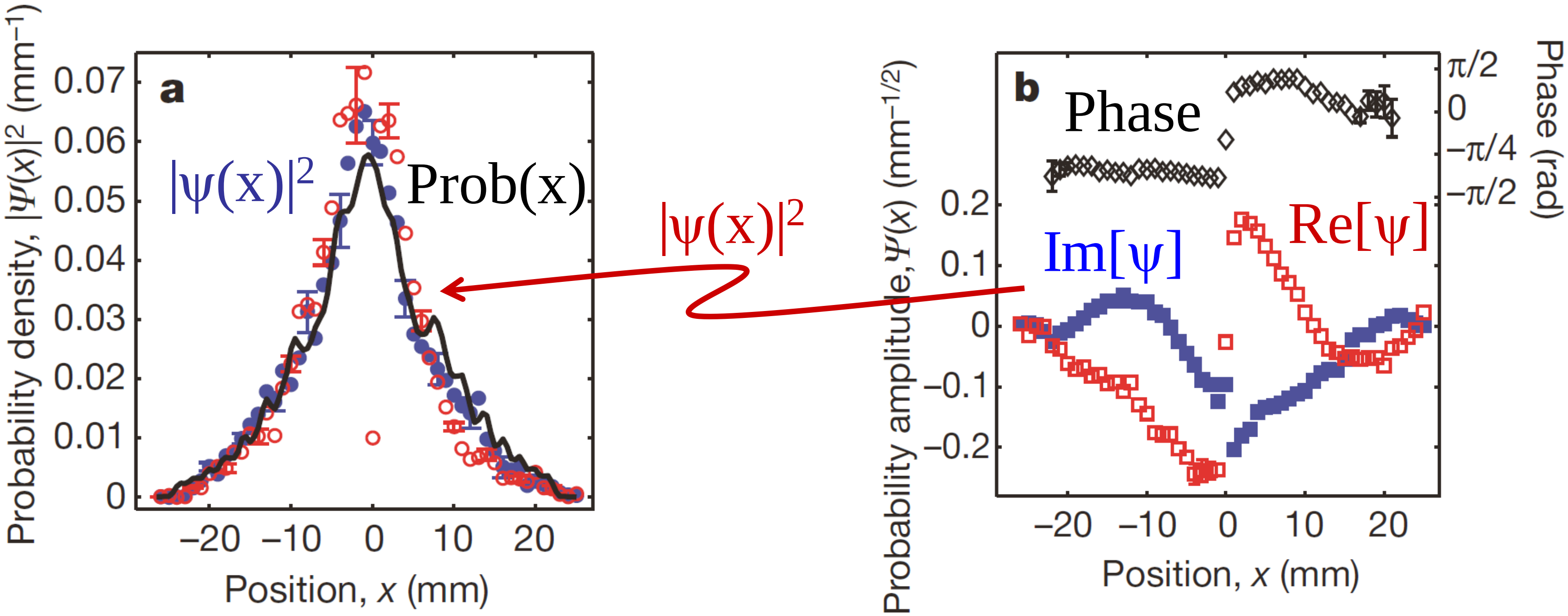
Testing another wavefunction shape

- Created new transverse wavefunction with a reverse bullseye filter



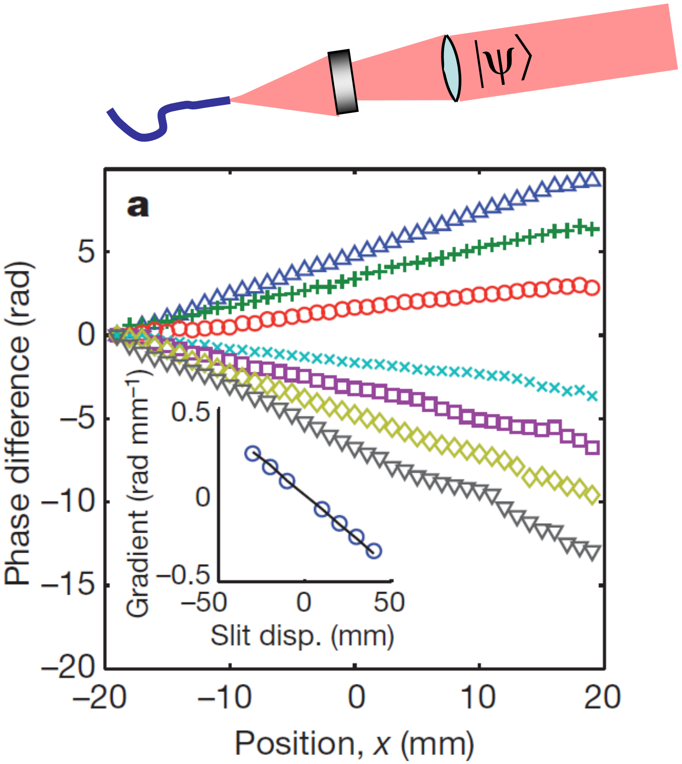
- Phase Discontinuity: Placed a glass square across half the wavefunction



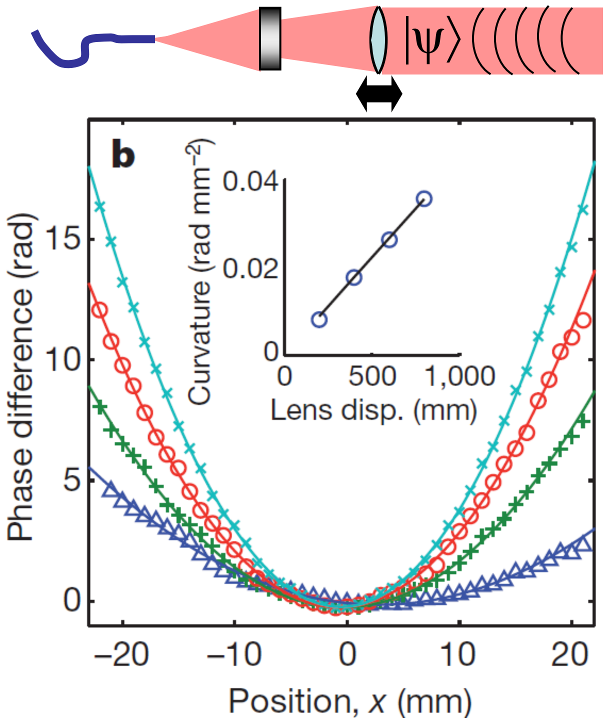


Testing other wavefunctions phase profiles

Phase Gradient

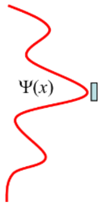


Phase Curvature

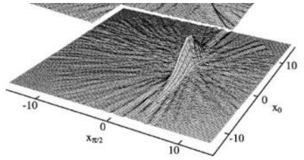


Why it is Direct

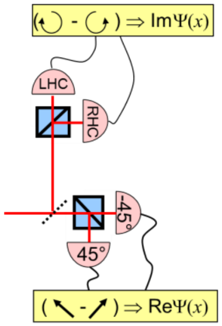
1. It is local - measures $\psi(x)$ at x



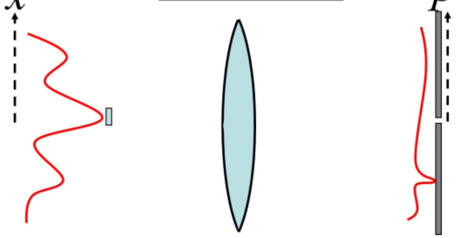
2. No complicated mathematical reconstruction

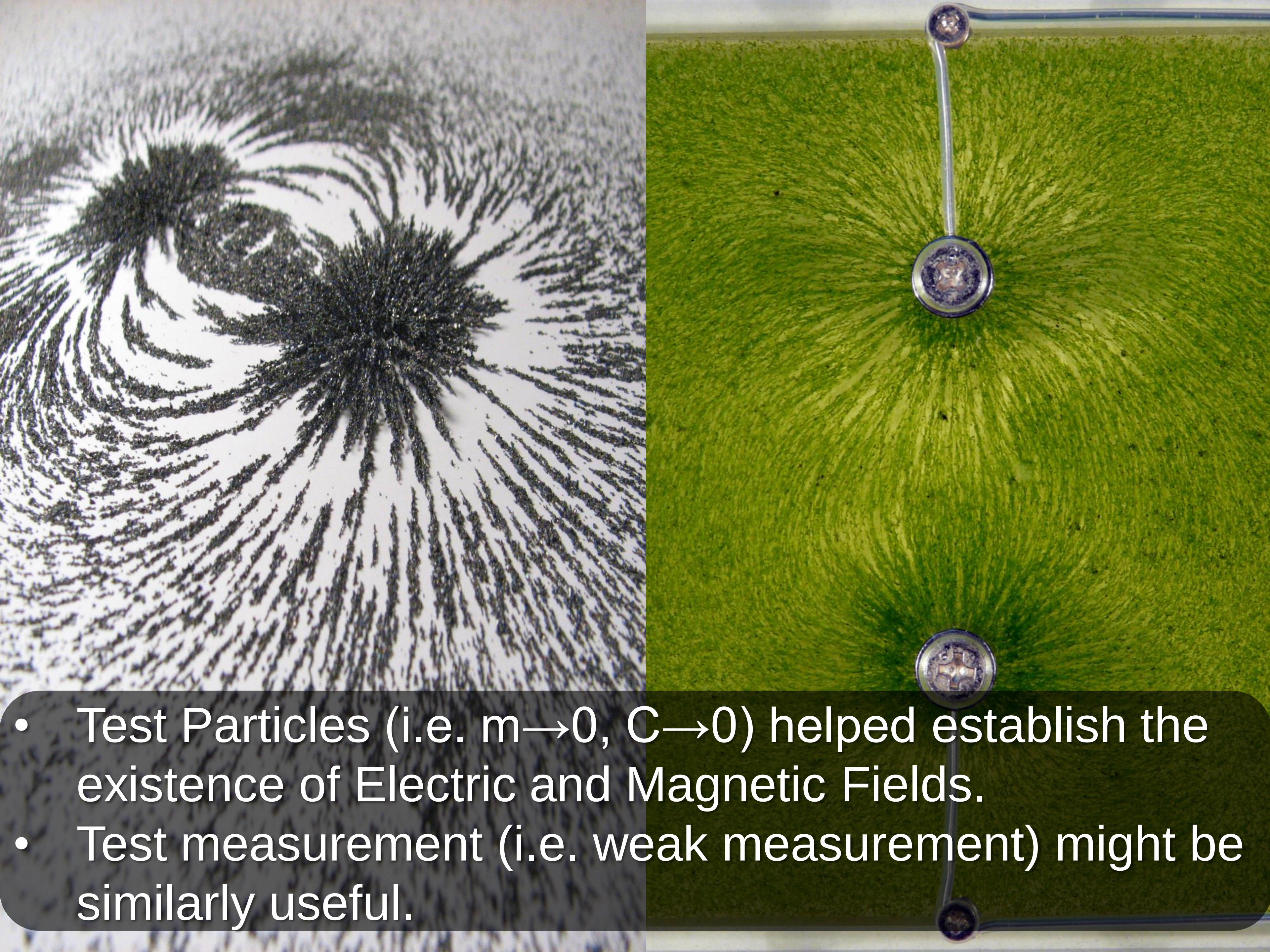


3. The value of $\psi(x)$ appears right on our measurement apparatus



4. The procedure is simple and general - measure x and then p





- Test Particles (i.e. m→0, C→0) helped establish the existence of Electric and Magnetic Fields.
- Test measurement (i.e. weak measurement) might be similarly useful.

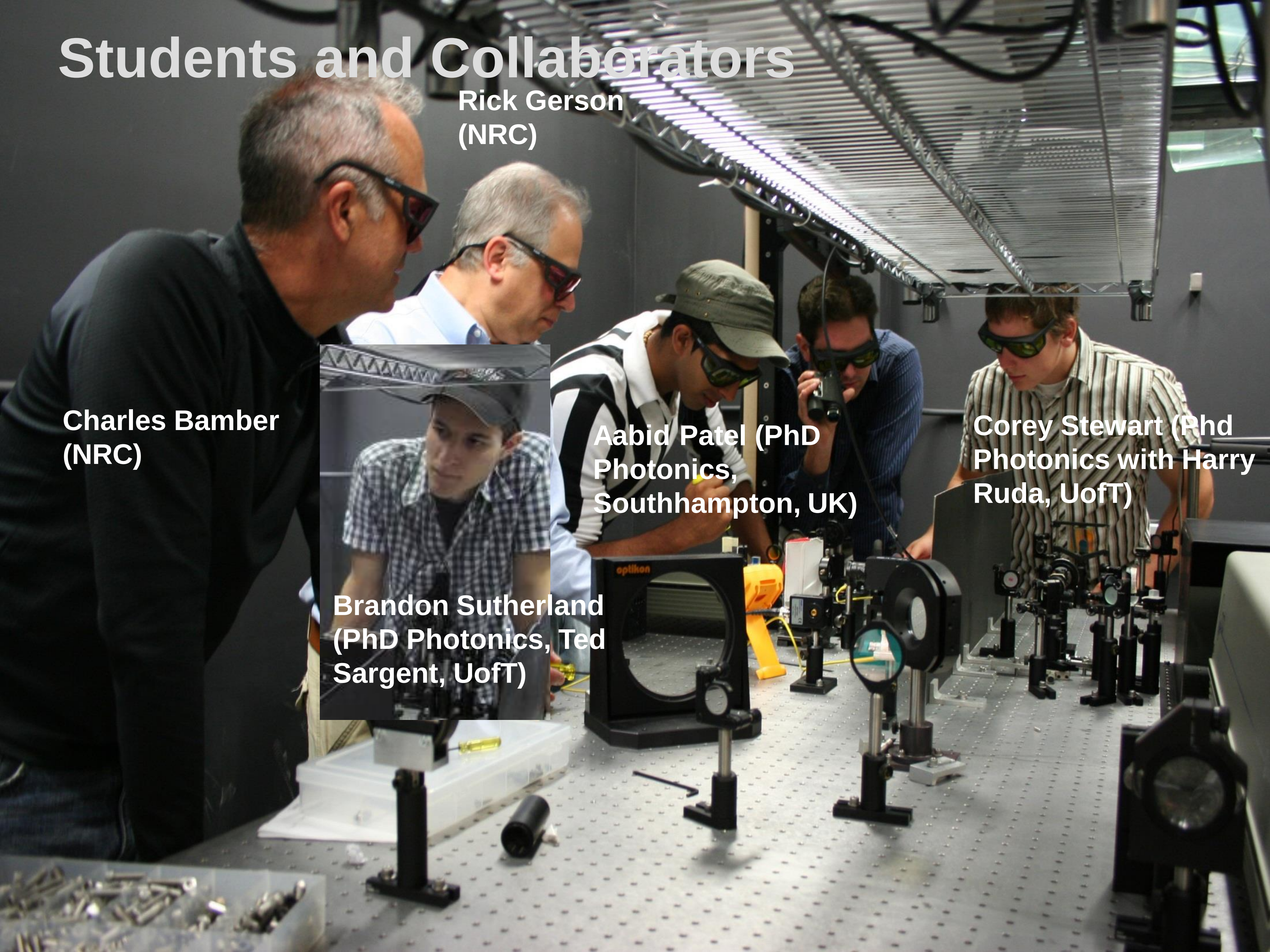
An operational definition of the wavefunction

- Currently there is no definition of the wavefunction.
- Clarity can come from “Operational” definitions of physical concepts.
  - i.e. the set of operations used in the lab to observe something.

Bridgman, P. The Logic of Modern Physics (1927).

“The wavefunction is the average result of a weak measurement of a variable followed by a strong measurement of the complementary variable”

Students and Collaborators



Rick Gerson
(NRC)

Charles Bamber
(NRC)

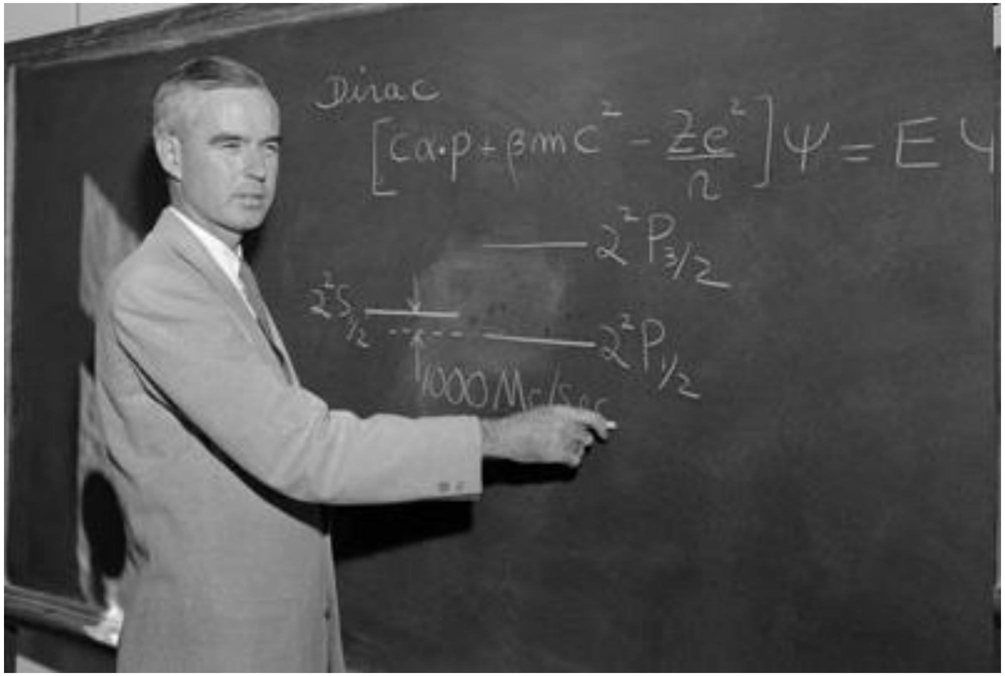
Aabid Patel (PhD
Photonics,
Southhampton, UK)

Corey Stewart (Phd
Photonics with Harry
Ruda, UofT)

Brandon Sutherland
(PhD Photonics, Ted
Sargent, UofT)

Conclusion

- Even though it may seem like a philosophical question, there has been progress (and more can be made!)
- The math is simple – undergraduates are probably asking the right questions (remember them!)
- Idea behind direct wavefunction measurement is universal
  - e.g. frequency-time photon wavefunction, electron spin state, entangled multiparticle states, etc.



Willis Lamb (Nobel Laureate). After writing Ψ on the blackboard, said to his class at Columbia:

Don’t worry about what this means, you’ll get used to it.

Recruiting undergrads, graduate students, and post-docs

www.photonicquantum.info for more info

Bob Boyd

Jacob Krich

Anne Broadbent

Ksenia Dolgaleva

Jeff Lundeen

At least one more