Applications of a nonlinear photon switch to Hardy’s Paradox and Bell-state determination

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Can we construct a two-photon gate?

Photons do not naturally interact: Great for transmission. Not so great for calculation.

**Proposed Solutions:**

- **Better materials by a factor of $10^{10}$**
  Absorptive nonlinearities (Franson), Resonance in Micro-structures (Gaeta, Walmsley)

- **Cavity Quantum Electrodynamics**
  Haroche, Kimble, Walther, Rempe

- **EIT**
  Harris, Scully, Lukin, Fleishhauer, Hau

- **Measurement-induced nonlinearities**
  Knill, Laflamme, Milburn, Franson, White, Zeilinger

- **Interference-enhanced nonlinearities**
  Exchange effects in atomic clouds (Franson), $\chi^{(2)}$ with interference (Steinberg)
Spontaneous Parametric Downconversion

- A pump photon is spontaneously converted into two lower frequency photons in a material with a nonzero $\chi^{(2)}$.

Momentum is conserved as well as energy:

$\mathbf{k}_s + \mathbf{k}_i = \mathbf{k}_{\text{PUMP}}$

$\omega_p = \omega_s + \omega_i$

$\phi_{\text{PUMP}} = \phi_s + \phi_i$
The Switch

The Absorptive Gate

- Phase chosen so that all photon pairs are “absorbed” into the pump beam

- On average < 1 photon per pulse
- One photon controls the transmission of the other beam
- The blue pump beam acts as a catalyst increasing SHG by a factor of $10^{10}$
The Phase Gate

- Set two-photon amplitudes so that they add up to give a phase-shifted output

\[
\begin{align*}
\alpha |00\rangle + \beta |10\rangle + \gamma |01\rangle + d |11\rangle \\
\alpha |00\rangle + \beta |10\rangle + \gamma |01\rangle + de^{ip/3} |11\rangle
\end{align*}
\]

Measurement of Phase-shift

- Turn one of the input beams into a Mach-Zehnder and insert gate in one arm.
Interference Fringes with and without Post-selection

![Graph showing interference fringes with and without post-selection. The graph plots coincidence counts against delay (fs) with error bars and includes quantum states |10⟩ and |11⟩.](image-url)
Variable Phase-Shifts

[Graph showing phase shifts versus pump phase shift with error bars]
Caveats

• Typically, optical quantum computing uses single photons

• Single-photons do not have a well defined phase

• Both the absorptive gate and the phase gate rely on interference and hence require input beams with a well defined phase

• In practice: Input beams = weak coherent states or SPDC beams

• Concept: We can’t know in advance whether the input beams contain a photon or not
Bell-state Analyzer

• Impossible to measure all four Bell-states with linear-optics

• Converts each Bell-state to a different basis state (i.e. |?\rangle \tau |HH\rangle)

• Insert interference-based phase-gate in place of CPHASE

• Works for Dense-Coding (send 2 bits with one photon)

• Doesn’t work for Teleportation

Interaction-Free Measurement


Bomb Absent:
Only detector C fires

Bomb Present:

<table>
<thead>
<tr>
<th>Detector</th>
<th>Prob.</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$\frac{1}{4}$</td>
<td>None</td>
</tr>
<tr>
<td>D</td>
<td>$\frac{1}{4}$</td>
<td>Present</td>
</tr>
<tr>
<td>Neither</td>
<td>$\frac{1}{2}$</td>
<td>Bang</td>
</tr>
</tbody>
</table>
Hardy’s Paradox


• Can we talk about the past in postselected QM?
• How should we interpret indirect quantum measurements?

### Outcome | Prob
--- | ---
$D_+ \text{ and } C_-$ | $1/16$
$D_- \text{ and } C_+$ | $1/16$
$C_+ \text{ and } C_-$ | $9/16$
$D_+ \text{ and } D_-$ | $1/16$
Explosion | $4/16$
Experimental Setup
Experimental Data

Switch: Vis=85.4%

Horizontal Pol. Mach-Zehnder: Vis=95.7%

Vertical Pol. Mach-Zehnder: Vis=97.4%
# Experimental Data

<table>
<thead>
<tr>
<th>Testing IFM+</th>
<th>If D+ clicks ⇒</th>
<th>Photon is in arm I-</th>
<th>96%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Photon is in arm O-</td>
<td>4%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Testing IFM-</th>
<th>If D- clicks ⇒</th>
<th>Photon is in arm I+</th>
<th>97%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Photon is in arm O+</td>
<td>3%</td>
</tr>
</tbody>
</table>

| Testing Switch | Rate of photon pairs in I+ and I- = 10.4 ± 0.33/5s |

| The Paradox | Rate of D+ and D- coincidences = 7.28 ± 0.41/5s |
## Weak Measurements

Aharonov, Albert, & Vaidman, PRL 60, 1351 ('88)

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Pointer Position Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal</td>
<td>Dirac Delta</td>
</tr>
<tr>
<td>Real</td>
<td>Width &lt;&lt; Change in Position</td>
</tr>
<tr>
<td>Weak</td>
<td>Width &gt;&gt; Change in Position</td>
</tr>
</tbody>
</table>

**Average position of pointer:**

\[
\text{Pointer}(X) = \exp[-(X - gA_W)^2 / \Delta X]
\]

\[
A_W = \frac{\langle \phi | A | \psi \rangle}{\langle \phi | \psi \rangle}
\]

**Average position of pointer:**

\[
\frac{E}{4} \quad \frac{1}{2} \quad \frac{3}{4} \quad F
\]

\[\Delta X \Delta P \geq \frac{\hbar}{2\pi}\]

⇒ small disturbance

⇒ little system – pointer entanglement

⇒ simultaneous measurement of different weak values

⇒ useful for investigating post-selected systems: Hardy's Paradox
Weak Measurements in Hardy’s Paradox
Resch & Steinberg, PRL 92, 130402 (2004)

# In Arm  | N(I⁻)  | N(O⁻)  
----------|--------|--------
N(I⁺)     | 0      | 1      
N(O⁺)     | 1      | -1     

1  0
Conclusions

• Interference-enhanced $\chi^{(2)}$ nonlinearities can be used to make absorptive and phase gates

• The phase-gate can be used to make a Bell-state analyzer useful for Dense-coding

• A single-photon level switch allows photons to annihilate each other with a high efficiency in Hardy’s Paradox

• We are now experimenting with weak measurements in Hardy’s Paradox.