Comment on "Linear optics implementation of weak values in Hardy's paradox"

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A recent experimental proposal [S.E. Ahnert and M.C. Payne, Phys. Rev. A **70**, 042102 (2004)] outlines a method to measure the weak value predictions of Aharonov in Hardy's paradox. This proposal contains flaws in the state preparation method and the procedure for carrying out the requisite weak measurements. We identify previously published solutions to some of the flaws.

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I. INTRODUCTION

Weak measurement is an extension of the von Neumann measurement model, in which the coupling of the measurement pointer to the measured system is small [1]. This prevents measurement-induced disturbance of the measured system and avoids collapse. Consequently, weak measurement is suitable for studying systems involving post-selection such as Hardy's paradox [2]. Hardy's paradox involves two overlapped Mach-Zehnder interferometers where each interferometer acts as an interaction-free measurement (IFM) [3] on the particle in the "inner" arm of the other interferometer. The paradoxical result is that occasionally both interaction-free measurements are positive and indicate the simultaneous presence of the two photons in the inner arms of the Mach-Zehnders, yet one never finds the photon-pair there when directly measured. Aharonov used weak measurement to find which Mach-Zehnder arms the photons were in, individually and as a pair, in the subensemble of systems for which the IFMs give their paradoxical result [4]. Unlike standard, strong quantum measurements, the results of these weak measurements, called "weak values", are consistent with each other and hence resolve the paradox. The resolution is nevertheless strange as it shows that there were negative one pairs of photons in the "outer" arms. Although there has been a proposal to test Aharonov's predictions in an ion system [5], as of yet there have been no experiments. This comment is on a recent experimental proposal by Ahnert and Payne for a linear optics implementation of Hardy's paradox and the aforementioned weak measurements [6]. We address two main problems in the proposal. First, the state preparation procedure does not produce the correct state for Hardy's paradox. Second, the outlined methods for conducting weak measurements of two-particle observables are not capable of measuring the weak values of the operators Aharonov investigated.

II. STATE PREPARATION PROCEDURE

In this section, we show that Ahnert and Payne's procedure for creating the necessary state with linear optics and post-selection does not work. The authors chose a polarization representation to encode the paths of the photons, where $|H\rangle$ represents the Mach-Zehnder inner path and $|V\rangle$ represents the outer. They aim to produce the nonmaximally entangled initial state $|\psi\rangle=(|HH\rangle+|HV\rangle+|VH\rangle)/\sqrt{3}$. They begin with two pairs of photons, each in the state $(|HH\rangle+|VV\rangle)/\sqrt{2}$, that enter the setup shown in Fig. 3 of Ref. [6]. Upon a detection at D', the authors claim the state collapses to the target initial state. Instead, their apparatus produces the density matrix $(|HH\rangle\langle HH|+|HV\rangle\langle HV|+|VH\rangle\langle VH|)/3$. The reason that there is no coherence between the terms in this state is because the apparatus cannot remove the which-path information remaining after the detection of a photon at D': $|HH\rangle$ results in no photons exiting the two polarizing beam splitters (PBS); $|HV\rangle$ results in a V photon exiting the upper PBS; and $|VH\rangle$ results in a V photon exiting the lower PBS. Tracing over the modes exiting the vertical ports of the two PBSs leaves a mixed state unsuitable for Hardy's paradox. In fact, this flaw is a relatively minor problem as there are other even simpler methods that require only one pair of photons and linear optics to produce the appropriate entangled state. Specifically, the target initial state $|\psi\rangle$ can be written as $a|\psi\phi\rangle+b|\psi^{\perp}\phi^{\perp}\rangle$ via a Schmidt decomposition. Beginning with a source that emits photon pairs in nonmaximally-entangled states of the form $a|HH\rangle+b|VV\rangle$ such as the one demonstrated in Ref. [7], the latter state can be produced by straightforward polarization rotations. A linear optics implementation of Hardy's paradox (but not the weak measurements) was proposed by Hardy in 1992 [8] and was recently demonstrated in Ref. [9] subsequent to the publication of Ref. [6].

III. TWO-PARTICLE WEAK MEASUREMENT

The aim of Ahnert and Payne was to suggest a feasible implementation of Aharonov's weak measurements. In Aharanov's work, the single-particle weak measurements are simply identical to the IFM results. It is the two-particle weak measurements that are most significant since they reveal a consistent resolution of Hardy's paradox. In this section, we address the two ways in which the paper proposes to do two-particle weak measurements.

First, we review Ahnert and Payne's single-particle weak measurements. They base much of their apparatus and theory on an earlier paper [10] which employs the arrival time of the photon as a measurement pointer similar to in Ref. [11]. They claim that their apparatus measures the operator $A_i = \gamma|V_i\rangle\langle V_i| + \varepsilon|H_i\rangle\langle H_i|$, where $i=1$ or 2 indicates the particle number. In contrast, Aharonov discussed the weak values of the polarization projectors $P_{H_i} = |H\rangle_i\langle H|_i$ or $P_{Vi} = |V\rangle_i\langle V|_i$. The weak value of A_i is related to the weak values Aharonov discussed by $\langle A_i\rangle_w = \gamma\langle P_{Vi}\rangle_w + \varepsilon\langle P_{Hi}\rangle_w$, where the subscript w indicates a weak value [4]. Using the identity $P_{Hi} + P_{Vi} = I$, one can derive the additional property of these weak values, $\langle P_{Vi}\rangle_w + \langle P_{Hi}\rangle_w = 1$, which along with $\langle A_i\rangle_w$, is sufficient to extract $\langle P_{Vi}\rangle_w$ and $\langle P_{Hi}\rangle_w$. The authors find that in Hardy's paradox $\langle A_i\rangle_w = \varepsilon$, from which we can infer that $\langle P_{Vi}\rangle_w = 1$ and $\langle P_{Hi}\rangle_w = 0$. These results agree with the subsequent IFMs (as they must) and indicate that, individually, each photon is in the inner arm (V polarization) of its respective interferometer.

In their first method of two-particle weak measurement, the authors represent the combined measurement of the two photons with an unconventional vector operator. The problem with this type of operator is that in the weak regime it measures quantities such as $\langle P_{V1}\rangle_w$ and $\langle P_{V2}\rangle_w$ as opposed to $\langle P_{V1}P_{V2}\rangle_w$, one of the Aharonov's two-particle weak values. The specific vector operator the authors propose to measure is $A_{12} = (\gamma,\gamma)|VV\rangle\langle VV| + (\gamma,\varepsilon)|HV\rangle\langle HV| + (\varepsilon,\gamma)|VH\rangle\langle VH| + (\varepsilon,\varepsilon)|HH\rangle\langle HH|$, which, again, is not any one of Aharonov's four two-particle projectors, such as $P_{VH} = |VH\rangle\langle VH|$, but a combination of all four. The operator can be expanded as a vector

$$A_{12} = \begin{pmatrix} \gamma|VV\rangle\langle VV| + \gamma|HV\rangle\langle HV| + \varepsilon|VH\rangle\langle VH| + \varepsilon|HH\rangle\langle HH|, \\ \gamma|VV\rangle\langle VV| + \varepsilon|HV\rangle\langle HV| + \gamma|VH\rangle\langle VH| + \varepsilon|HH\rangle\langle HH| \end{pmatrix} \quad (1)$$

$$= \begin{pmatrix} (\varepsilon|H_2\rangle\langle H_2| + \gamma|V_2\rangle\langle V_2|)(|H_1\rangle\langle H_1| + |V_1\rangle\langle V_1|), \\ (\varepsilon|H_1\rangle\langle H_1| + \gamma|V_1\rangle\langle V_1|)(|H_2\rangle\langle H_2| + |V_2\rangle\langle V_2|) \end{pmatrix} \quad (2)$$

$$= (\varepsilon|H_2\rangle\langle H_2| + \gamma|V_2\rangle\langle V_2|, \varepsilon|H_1\rangle\langle H_1| + \gamma|V_1\rangle\langle V_1|) \quad (3)$$

$$= (A_2, A_1). \quad (4)$$

On the surface, A_{12} appears to be a two-particle operator representing the measurement of some joint property of the two photons. In fact, Eq. (4) shows that A_{12} can be re-expressed as a vector of two separate single-particle operators, representing independent measurements of A_i on each photon. Contrast this with the correct combination, $A_1 \otimes A_2$, which Aharonov employed in operators such as $|HV\rangle\langle HV|$. Therefore, their proposal, built upon the operator A_{12}, is insufficient to measure any of the two-particle weak values in Hardy's paradox. For example, the weak value for $\langle A_{12}\rangle_w = (\langle A_2\rangle_w, \langle A_1\rangle_w) = (\varepsilon,\varepsilon)$ {Eq. (23) in [6]} simply contains the weak values from the single-particle weak measurements reviewed above. If one incorrectly interprets this result as the weak value for the location of the photon pair then one would conclude that the photons were simultaneously in the inner arms (had vertical polarizations). This is in direct contradiction to Aharonov's prediction of 0 for the weak value $\langle P_{V1}P_{V2}\rangle_w$, a prediction which necessarily must concur with Hardy's paradox [4] in that the photon pair is never found in the inner arms simultaneously.

Designing a linear optics experiment to implement the single-particle weak measurements was never a major hurdle for Hardy's paradox. Measurement of a single-particle weak value is straightforward when the pointer variable is another degree of freedom in the quantum system. Examples include a Stern-Gerlach device for measuring spin by coupling to the transverse momentum of the particle, or the optical analogy with polarization [12]. Difficulties begin when one means to measure joint properties of multiple particles. As with strong quantum measurements, weak values do not obey a product rule, (i.e., $\langle AB\rangle_w \neq \langle A\rangle_w\langle B\rangle_w$ [4]), so that multiparticle weak values cannot be calculated from single-particle ones. Instead, they must be measured. In the present example, one has to weakly measure projectors such as $|HH\rangle\langle HH|$. If one follows the original approach to weak measurement, based on von Neumann system-pointer interactions, nonlinear optical interactions at the two-photon level are required to measure this projector. Recently, two of us devised a method that avoids this obstacle [13] and is ideally suited for linear optics. In that work, we show how one can indirectly extract joint weak values by studying the correlations between two single-particle weak measurements. This procedure has since been extended and simplified [14]. In summary, a serious flaw in Ahnert and Payne's proposal is that they do not outline any such indirect method for weakly measuring two-particle joint properties while, at the same time, their apparatus is incapable of directly measuring these properties.

Their second method of two-particle weak measurement consists of inserting detectors in the apparatus before the post-selection. In the penultimate paragraph of their paper, the authors assert that one can measure three of the four two-particle weak values with this method. However, the detectors would collapse each of the two photons to either H or V polarization and strongly disturb the subsequent post-selection. This is a strong, intrusive measurement and is exactly the situation that weak measurement is designed to circumvent. Furthermore and not surprisingly, these measurements give the strong measurement results, not the weak ones.

IV. CONCLUSION

In conclusion, because of these flaws the paper does not outline a feasible linear optics implementation of weak measurements in Hardy's paradox. However, the main obstacles to a linear optics implementation—state preparation and two-particle weak measurements—have been described and solved in other works in the literature. Consequently, there was already a clear way to measure weak values in Hardy's paradox using only linear optics.

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[1] Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988); Y. Aharonov and L. Vaidman, J. Phys. A **24**, 2315 (1991); Y. Aharonov and L. Vaidman, Phys. Rev. A **41**, 11 (1990).
[2] L. Hardy, Phys. Rev. Lett. **68**, 2981 (1992).
[3] A. Elitzur and L. Vaidman, Found. Phys. **23**, 987 (1993).
[4] Y. Aharonov, A. Botero, S. Popescu, B. Reznik, and J. Tollaksen, Phys. Lett. A **301**, 130 (2001).
[5] K. Mølmer, Phys. Lett. A **292**, 151 (2001).
[6] S. E. Ahnert and M. C. Payne, Phys. Rev. A **70**, 042102 (2004).
[7] A. G. White, D. F. V. James, W. J. Munro, and P. G. Kwiat, Phys. Rev. A **65**, 012301 (2002).
[8] L. Hardy, Phys. Lett. A **167**, 17 (1992).
[9] W. T. M. Irvine, J. F. Hodelin, C. Simon, and D. Bouwmeester, e-print quant-ph/0410160.
[10] S. E. Ahnert and M. C. Payne, Phys. Rev. A **69**, 042103 (2004).
[11] D. R. Solli, C. F. McCormick, R. Y. Chiao, S. Popescu, and J. M. Hickmann, Phys. Rev. Lett. **92**, 043601 (2004).
[12] N. W. M. Ritchie, J. G. Story, and R. G. Hulet, Phys. Rev. Lett. **66**, 1107 (1991).
[13] K. J. Resch and A. M. Steinberg, Phys. Rev. Lett. **92**, 130402 (2004).
[14] K. J. Resch, J. Opt. B: Quantum Semiclassical Opt. **6**, 482 (2004); J. S. Lundeen and K. J. Resch, Phys. Lett. A **334**, 337 (2005).