A double-slit ‘which-way’ experiment addressing the complementarity–uncertainty debate

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An interference pattern appears when it is impossible to tell which of two paths a particle took to a screen. Therefore, performing a which-way measurement (WWM) to determine which path it took necessarily destroys this pattern. In his debates with Einstein, Bohr [1] attributed this to the Heisenberg uncertainty principle: distinguishing two positions a distance s apart gives the particle a random momentum transfer $q \sim \hbar/s$. In 1991, Scully, Englert and Walther (SEW) [2] proposed a WWM that, they claimed, entails no momentum transfer. In response, Storey, Tan, Collett and Walls (STCW) [3] proved a theorem that, they claimed, confirms Bohr’s standpoint. Recently one of us [4] proposed a way to observe directly the momentum transfer $q$, via the weak-valued probability distribution $P_{\text{wv}}(q)$. As in STCW’s theorem, the width of $P_{\text{wv}}(q)$ is at least $\hbar/s$. But for a WWM like SEW’s (with nonclassical momentum transfer [5]) $P_{\text{wv}}(q)$ can take negative values, and its variance can be zero. Here we measure $P_{\text{wv}}(q)$ in an experiment akin to SEW’s but using photons. We find that its width is clearly greater than $\hbar/s$, but its variance is consistent with zero, thus reconciling the two standpoints.

SEW’s proposal purports that complementarity (manifested as the destruction of interference by which-way information) is an independent, fundamental principle of quantum mechanics, rather than a consequence of the uncertainty principle, as implied by Bohr [1] and also Feynman [6]. Following further debate [7], it was pointed out [5] that the quantitative results of both groups were correct, and not in conflict — they were simply using different concepts of momentum transfer, which agreed only for classical transfers (i.e. random kicks) like those of Bohr and Feynman. SEW showed that a single-slit wavefunction would be unchanged by their WWM. The theorem of STCW, on the other hand, meant that if the initial state were a momentum eigenstate then after any WWM the final momentum distribution would have a width [8] of at least $\hbar/s$ [9]. Although these calculations of SEW and STCW are consistent, it is unsatisfying that their physical predictions require experiments (with a single-slit wavefunction and momentum eigenstate respectively) that are incompatible with each other and with the double-slit experiment that they are supposed to illuminate. By contrast, the weak-valued probability distribution $P_{\text{wv}}(q)$ can (indeed, must) be observed while carrying out the original double-slit experiment. Moreover, it can be observed directly, in the sense that the distribution can be derived via a simple prescription, with no reference to quantum physics, from measurements a classical physicist would understand. And finally, its mean and variance exactly reflect the change in the mean and variance of the momentum distribution that occurs as a result of the WWM [10]. The experiment we report is the first to address the question of momentum transfer by WWMs in a double-slit apparatus, as treated by Bohr, Feynman, SEW and STCW. In particular, the elegant experiment by Rempe and co-workers [11] was not relevant to this issue. As they say: “In our experiment, no double slit is used and no position measurement is performed, so that the results of [Reference 9] do not apply.”

1. Theory.

The drastic disturbance of the state in a ‘strong’ or projective measurement [12] can be avoided by using weak measurements, at the price of obtaining an imprecise measurement result. A weak value $\langle X_w \rangle$ is the ensemble mean value from weak measurements of some quantity $X$ on identically prepared systems. This differs from the strong value if one obtains the weak value by post-selecting only those results for which a later strong measurement reveals the system to be in state $|\phi\rangle$. For an initial state $|\psi\rangle$, the weak value evaluates to $\langle X_w \rangle_\psi = \text{Re} \frac{\langle \phi | U \hat{X} | \psi \rangle}{\langle \phi | U | \psi \rangle}$. (1)

Here we have allowed for some evolution $\hat{U}$ after the weak measurement. This result is interesting because it may lie outside the range of eigenvalues of $\hat{X}$ [13], a prediction that was quickly verified experimentally [14]. Weak values have now been used to analyse a great variety of quantum phenomena [15–23].
A weak-valued probability (WVP) arises when the operator $\hat{X}$ is a projector $\hat{\pi}$. Without post-selection, the weak (or strong) value is $\langle \psi | \hat{\pi} | \psi \rangle$, which is a true probability. However, the post-selected WVP $\rho_\psi \langle \pi_w \rangle_\psi$ can lie outside the range $[0, 1]$. Reference [25] applied this concept to momentum transfer in WWMs. The basic idea is to find the weak value for the momentum projector

$$\hat{\pi}(p_i) = \int_{p_i - \Delta/2}^{p_i + \Delta/2} dp |p\rangle \langle p|$$

post-selected on measuring the final particle momentum $p_f$. Here $p_i$ is chosen by the experimenter, and $\Delta$ should be $\ll \hbar/s$, the interference pattern fringe spacing. When applying Eq. (1), the unitary evolution must be replaced by an operation describing the measurement of the particle by the WWM device [4, 10]. Since $p_i \langle \pi_w(p_i) \rangle_\psi$ is the WVP for the initial momentum to lie within $\Delta/2$ of $p_i$, given that the final momentum is $p_f$, we write it as the conditional WVP $P_wv_p(p_i|p_f)$.

If there is no WWM then $P_wv(p_i|p_f) = 1$ if $|p_i - p_f| < \Delta/2$ and 0 otherwise. Any deviation from this represents a momentum disturbance. This can be quantified by calculating a WVP distribution for the momentum transferred:

$$P_wv(q) = \sum_{p_i} P_wv(p_i; p_i + q),$$

where we have substituted $p_i + q$ for $p_f$ in $P_wv(p_i; p_f) = P_wv(p_i|p_f) \cdot P(p_f)$. Here $P(p_f)$ is the probability distribution for measuring the final momentum to be $p_f$. The sum in Eq. (3) is for $p_i = n\Delta$, with $n$ an integer, and means repeating the experiment for these values of $p_i$.

For WWMs that produce random momentum kicks [5], $P_wv(q)$ is guaranteed positive. However, in some cases $P_wv(q)$ may take negative values, indicating a nonclassical momentum transfer [5]. Nevertheless, it always integrates to unity, and must have a width [8] of at least $\hbar/s$, which reflects the theorem of STCW. On the other hand, in the SEW scheme where the WWM has no effect on a single-slit wavefunction, the mean and variance of $P_wv(q)$ are exactly zero (in the limit $\Delta \to 0$), reflecting the no-disturbance calculation of SEW [25].

2. Experiment.

The experimental apparatus is shown in Fig. 1. Since photons are non-interacting particles, we can use a large ensemble simultaneously prepared with the same wavefunction. It follows that the intensity distribution of the light is proportional to the probability distribution for each photon. Treating the photons as particles, a classical physicist would analyze the experiment in terms of particle trajectories [24]. In this model, the transverse motion of the photon is that of a free non-relativistic particle of mass $m = \hbar/c\lambda$.

The photon ensemble is produced by a $\lambda = 633\text{nm}$ HeNe laser that illuminates a double-slit aperture with a slit width of $40\mu\text{m}$ and a center to center separation of $s = 80\mu\text{m}$. We call the long axis of the slits $y$ and the axis joining their centers $x$. We use $f = 1\text{m}$ focal-length lenses to switch back and forth between position and momentum space for the photons. These can be treated as impulsive harmonic potentials in the classical particle picture. One metre after the first lens, the photon’s $x$-position $x_i$ becomes equal to $(f/c) \cdot (p_i/m)$, where $p_i$ is its initial momentum at the double-slit. Consequently, in the $x$-direction the intensity distribution is of the expected double-slit interference pattern with a fringe spacing of $8.2 \pm 0.1\text{mm}$. In the $y$-direction, the intensity distribution is Gaussian with a $1/e^2$ half-width $\sigma = 1.01 \pm 0.01\text{mm}$.

We weakly measure $\hat{\pi}(p_i)$ by coupling it to the $y$ degree of freedom of the photon, which we will later measure directly. An optically flat glass sliver with a width of $\delta = 1.77 \pm 0.02\text{mm}$ in the $x$-direction and a thickness of $1.00 \pm 0.25\text{mm}$ is placed at $x_i$ and tilted so that it creates a $y$-displacement of $0.14 \pm 0.01\text{mm}$ ($< \sigma$ ensures weakness) in a range of momenta $\Delta$ centered around $p_i$. Fine adjustment of the tilt ensured that upon passing through the glass the photons accumulated no additional phase (modulo $2\pi$).

To implement the WWM we must switch back to position space with a second $1\text{m}$ lens, in essence imaging the slits. In front of the image of one of the slits we place a $\lambda/2$ waveplate, which rotates the polarization so that it is orthogonal to that of the other slit. That is, the photon polarization carries the WWM result, destroying the double-slit interference. Since the waveplate would not alter a single-slit wavefunction emerging from either slit, this WWM is of the type SEW considered.

A third lens transforms back into momentum space, so that finally $x_f = (f/c) \cdot (p_f/m)$. Here we record the intensity distribution with a movable CCD camera in an $x$-$y$ region of size $27.5\text{mm} \times 2.7\text{mm}$. This was done for $x_i = n\delta$ for $n$ running from $-7$ to $7$ as well as the case in which the glass sliver, and hence weak measurement, is absent.

3. Results.

The final intensity distribution in the $x$-direction is a single-slit diffraction pattern proportional to $P(p_f)$. We determine the conditional WVP $P_wv(p_i|p_f)$ by finding the mean displacement in the $y$-direction of the intensity distribution at $x_f$ while the glass sliver is at $x_i$. This displacement is normalized by the displacement induced by the glass sliver at $x_i = 0$, measured with no WWM. The location of zero-displacement on the CCD is determined by using the intensity distribution with the glass sliver absent. As an example, in Fig. 2 we show $P_wv(p_i|p_f)$ for $p_i = -1.8\text{mm} (mc/f)$ along with a theoretical curve with no free parameters. Notice that the WVP becomes negative for certain values of $p_f$. We sum the conditional probabilities for all fifteen $p_i$ according to Eq. 3 to obtain the unconditional WVP $P_wv(q)$ of a momentum transfer $q$, plotted in Fig. 3 along with a parameterless theoretical curve. This shows that with the SEW-type WWM $P_wv(q)$ is nonzero outside the range $[-\hbar/s, \hbar/s]$, substantiating SCTW’s theorem.
Nonetheless, we also expect $P_{\psi\psi}(q)$ to have zero variance, consistent with SEW’s conclusions. Unfortunately, the discontinuity of the double-slit wavefunction $\psi(x)$ results in a theoretical $P_{\psi\psi}(q)$ that falls off slowly enough to require apodization to evaluate the variance [10]. Since experimental data sets are finite we instead calculate the variance over the range $[-\sigma_{\text{max}}, \sigma_{\text{max}}]$ without apodization. The theory predicts a variance that diverges as a function of $\sigma_{\text{max}}$, oscillating between positive and negative values as it must for the apodized variance to evaluate to zero. This is plotted in the inset of Fig. 3, along with the experimental variance, which agrees well. This is the experimental signature of the absence of momentum-disturbance (in the SEW sense).

SEW also considered the retrieval of interference through the use of a quantum eraser. In this case, what happens to the photons that experienced a momentum transfer? Photons transmitted through a 45° polarizer, our quantum eraser, form a double-slit interference pattern, whereas the −45° photons form the opposite interference pattern, having had a phase-shift induced by the eraser. In the supplementary material we plot $P_{\psi\psi}(p_i | p_f)$ with $p_i = -1.8\text{mm}(\text{mc}/f)$ for both polarizer settings, which shows that the interference-destroying momentum transfer now appears solely in the the −45° light.

4. Conclusion.

These observations clearly demonstrate that momentum-transfer probabilities defined by weak measurements, which can be extracted without the introduction of any quantum-mechanical formalism or interpretation, are consistent with the predictions of both SEW and STCW. Due to the possibility of negative weak-valued probabilities, there is no longer any incompatibility between the observations that which-path measurements induce some momentum transfers greater than $h/s$ and that the variance of the induced momentum transfer vanishes.

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References

[8] When we say a distribution has width of at least $\sigma$ we mean that it is nonzero somewhere outside the interval $[-\sigma, \sigma]$.


[25] Moreover, it was shown in Ref. [9] that no disturbance of the single slit pattern implies no broadening of the double-slit *envelope*.

**Supplementary Information accompanies the paper on www.nature.com/nature.**

Figure 1 Diagram of apparatus. After the photons are prepared in the initial state by a polarizing beam-splitter (PBS) and a double-slit aperture, the experiment can be divided into three stages: The weak measurement of the initial momentum $p_i$, the which-way measurement consisting of a half-wave plate (HWP) that rotates the polarization of one slit by $90^\circ$, and the strong measurement of $p_f$ by a CCD camera. Below are shown representative intensity distributions at the double-slit, after the glass sliver, upon reimaging the slits, and at the CCD.

Figure 2 Weak Measurement results for $p_i = -1.8 \text{mm} \cdot (mc/f)$. The dots indicate the $y$-displacement of the intensity distribution at each $x$-position, and hence $p_f$, on the CCD. The thin solid line is a parameterless theoretical curve. The solid black rectangles indicate the region of weak measurement (bounded by $p_i \pm \Delta/2$). This displacement is proportional to the weak-valued probability $P_{wv}(p_i|p_f)$.

Figure 3 The weak-valued probability distribution for the momentum transfer $P_{wv}(q)$. The dots are experimental points and the thin solid line is a parameterless theoretical curve. The inset is the variance of experimental data (solid circles) in the range $[-q_{\text{max}}, q_{\text{max}}]$ as a function of $q_{\text{max}}$ along with the theoretical prediction (solid line).
Supplementary Discussion

A quantum eraser [1] is a measurement on the WWM apparatus that retrieves the double-slit interference pattern destroyed by the WWM. That is, one sorts the particles into bins according to the results of a measurement performed in a conjugate basis to the one that carries the WWM result. Even though the entire set of particles will still not form an interference pattern, the subset of particles in each bin will. For a WWM with classical momentum transfer, the different bins contain identically shaped interference patterns, but are shifted in the $x$-direction by varying amounts [2, 3]. By contrast, for a WWM such as that of SEW [4], the interference patterns in the different bins all have the same envelope; only the phases of the patterns differ [5].

In this section, we investigate the change a quantum eraser makes to the nonclassical momentum transfer we presented in Fig. 3 of the paper. Since our WWM result is carried in the Horizontal/Vertical basis of the photon polarization, we implement a quantum eraser with a polarizer in the $45^\circ$/$-45^\circ$ basis. This sorts the photons into two bins, $45^\circ$, which form the original interference pattern (fringes) and $-45^\circ$, which form one with the opposite phase (antifringes). In Supplemental Fig. 1, we plot $P_{wv}(p_i|p_f)$ with $p_i = -1.8\text{mm}-(mc/f)$ for both polarizer settings, along with the measured interference patterns. For the photons transmitted at $45^\circ$, the data shows that, to a good approximation, $P_{wv}(p_i|p_f) = 1$ if $|p_i - p_f| < \Delta/2$ and 0 otherwise, indicating that there is no momentum transfer. On the other hand, for the $-45^\circ$ photons, $P_{wv}(p_i|p_f)$ is substantially different from zero even for $p_f$ outside the range $p_i \pm \Delta/2$, reflecting a momentum disturbance. This result is found for all values of $p_i$, showing clearly that the momentum transfer only appears in the $-45^\circ$ photons. Since these are the anti-fringe photons, this indicates an intimate connection between the nonclassical momentum transfer and the phase between the slits induced by the quantum eraser.

References


Supplementary Figure 1  The weak-valued probability $P_{\text{ww}}(p_i|p_f)$ for $p_i = -1.8\text{mm} \cdot (\text{mc}/f)$ with a quantum eraser consisting of a) a 45° polarizer and b) a -45° polarizer, both placed after the which-way measurement. The dots indicate the y-displacement of the intensity distribution at each x-position, and hence $p_f$, on the CCD. The diamonds indicate the intensity at each $p_f$. The vertical lines indicate $p_i \pm \Delta/2$, the region of the weak measurement.